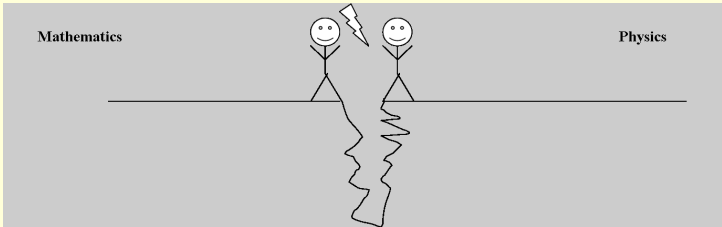
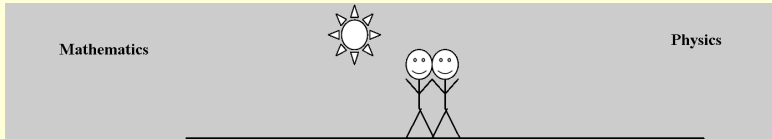


Using Geometric Reasoning to Teach Vector Calculus in Mathematics and Physics

Tevian Dray & Corinne Manogue



Mathematics vs. Physics



Vector Calculus Bridge Project:

<http://math.oregonstate.edu/bridge>

- Differentials (*Use what you know!*)
- Multiple representations
- Symmetry (*adapted bases, coordinates*)
- Geometry (*vectors, div, grad, curl*)
- Online text (<http://math.oregonstate.edu/BridgeBook>)

Paradigms in Physics Project:

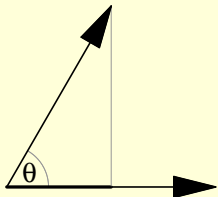
<http://physics.oregonstate.edu/portfolioswiki>

- Redesign of undergraduate physics major (*18 new courses!*)
- Active engagement (*300+ documented activities!*)



Dot Product

Tell me something you know about the dot product.
Write your answer on your small whiteboard.



Projection:

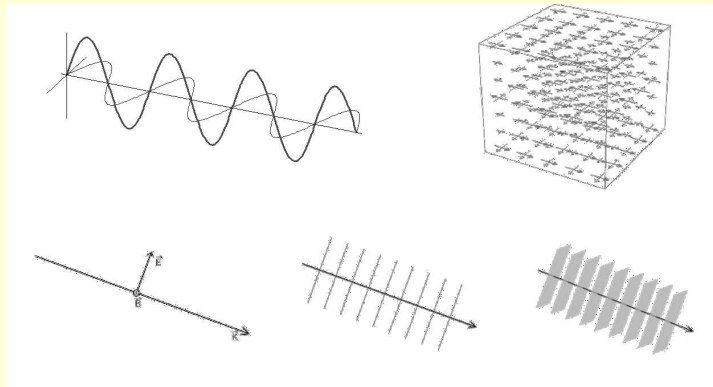
$$\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta$$

$$\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y$$

Compare and Contrast

- On your medium whiteboards, construct a square grid of points, approximately 2 inches apart, at least 7×7 .
- I will draw an origin and a vector $\vec{\mathbf{k}}$ on your grid.
- For every point on your grid, imagine drawing the position vector $\vec{\mathbf{r}}$ to that point; calculate $\vec{\mathbf{k}} \cdot \vec{\mathbf{r}}$.
- Connect the points with equal values of $\vec{\mathbf{k}} \cdot \vec{\mathbf{r}}$.

Plane Wave Representations



What are Functions?

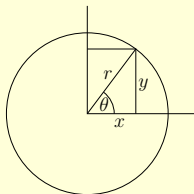
Suppose the temperature on a rectangular slab of metal is given by

$$T(x, y) = k(x^2 + y^2)$$

where k is a constant. What is $T(r, \theta)$?

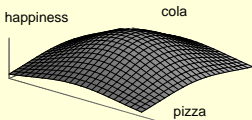
A: $T(r, \theta) = kr^2$

B: $T(r, \theta) = k(r^2 + \theta^2)$

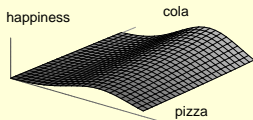


Pizza

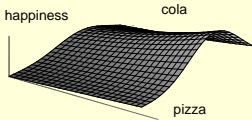
You like pizza and you like cola. Which of the graphs below represents your happiness as a function of how many pizzas and how much cola you have if *there is no such thing as too many pizzas and too much cola*?



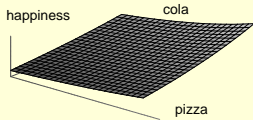
A



B



C



D

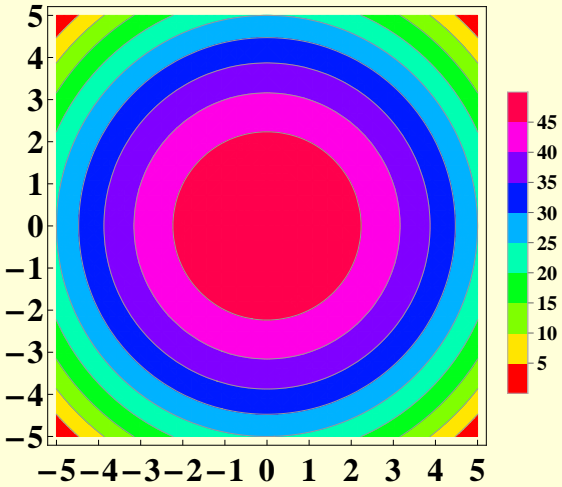
Charge Density

- Please stand up.
- Each of you represents a charge.
- Make a linear charge density.

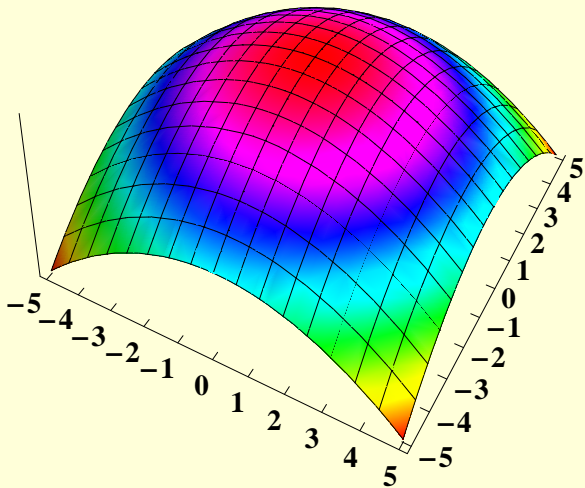
Table

0	9	16	21	24	25	24	21	16	9	0
9	18	25	30	33	34	33	30	25	18	9
16	25	32	37	40	41	40	37	32	25	16
21	30	37	42	45	46	45	42	37	30	21
24	33	40	45	48	49	48	45	40	33	24
25	34	41	46	49	50	49	46	41	34	25
24	33	40	45	48	49	48	45	40	33	24
21	30	37	42	45	46	45	42	37	30	21
16	25	32	37	40	41	40	37	32	25	16
9	18	25	30	33	34	33	30	25	18	9
0	9	16	21	24	25	24	21	16	9	0

Level Curves



Graph



Tell me something you know about the gradient.

Write your answer on your small whiteboard.

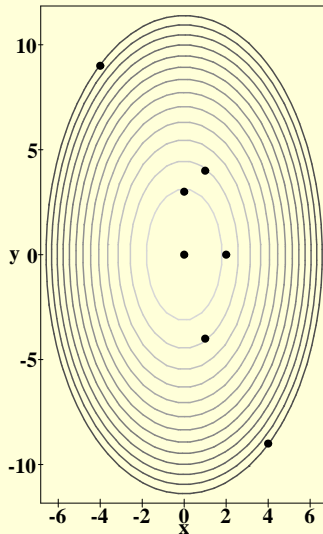
- The gradient points in the steepest direction.
- The magnitude of the gradient tells you how steep.
- $\vec{\nabla} f = \frac{\partial f}{\partial x} \hat{\mathbf{x}} + \frac{\partial f}{\partial y} \hat{\mathbf{y}} + \dots$

The Hill

Suppose you are standing on a hill. You have a topographic map, which uses rectangular coordinates (x, y) measured in miles. Your global positioning system says your present location is at one of the points shown. Your guidebook tells you that the height h of the hill in feet above sea level is given by

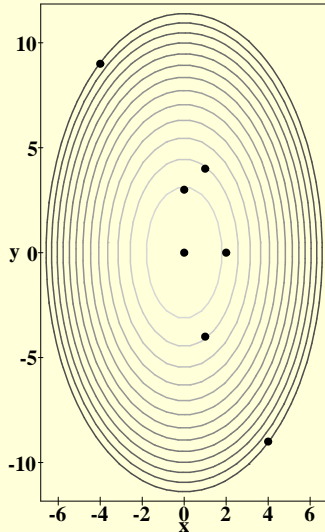
$$h = a - bx^2 - cy^2$$

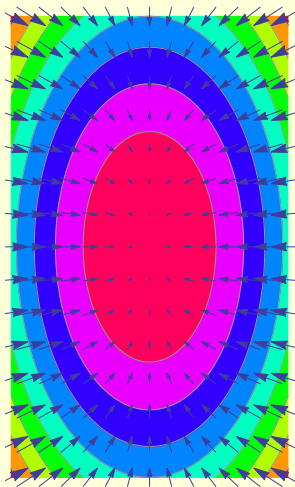
where $a = 5000$ ft, $b = 30 \frac{\text{ft}}{\text{mi}^2}$,
and $c = 10 \frac{\text{ft}}{\text{mi}^2}$.



The Hill

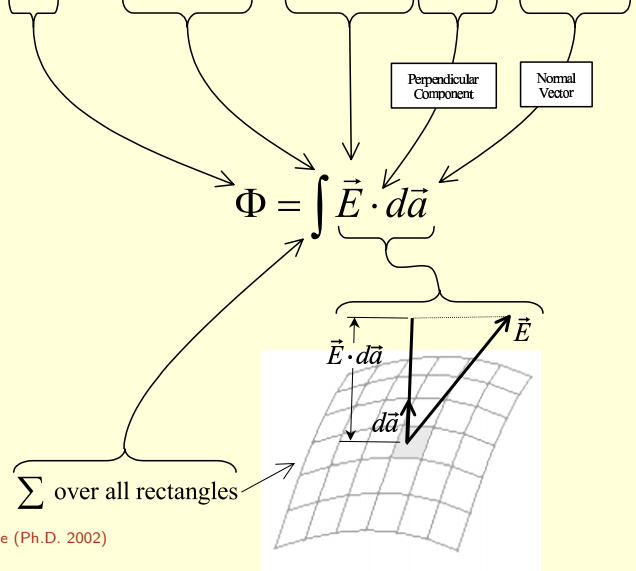
*Stand up and close your eyes.
Hold out your right arm in the
direction of the gradient where
you are standing.*





SUMMARY

Flux is the total amount of electric field through a given area.



Kerry Browne (Ph.D. 2002)

Geometry, geometry, geometry...



Tevian Dray & Corinne A. Manogue

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