Active Engagement: Lessons learned from the Paradigms and Bridge projects

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Get Ready:

Please sit with one or two other partners.

For each group, please pick up:

- 1 small whiteboard;
- 1 whiteboard pen;
- 1 high-tech eraser (e.g. tissue/napkin);

Teaching

Good teaching is like picking up someone else's baby.

Using the Quaternions to Implement Rotations

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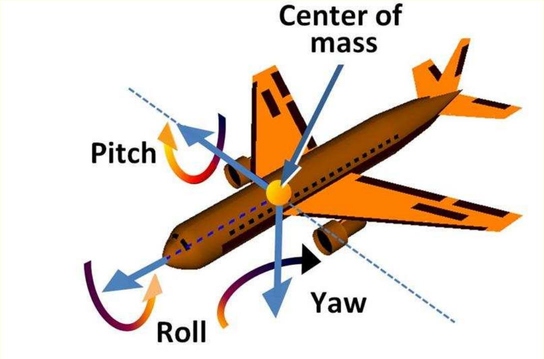
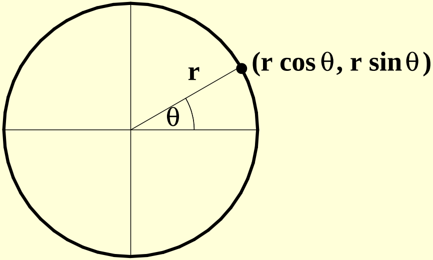
Introduction



3-d rotations: Aeronautics, robotics, computer graphics, ...

New Content: Use quaternions to implement rotations

Trigonometry



Polar coordinates: $x = r\cos\theta;\ y = r\sin\theta.$

Circle Trig

Things to consider:

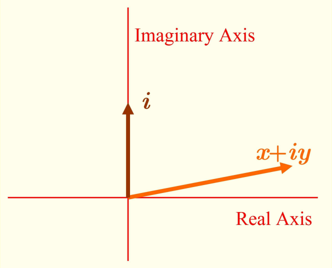
- Whenever possible, base your instruction on what is known about incoming student resources.
- Example: Dr. Emily Smith (OSU 2016) showed that many upper-division physics students know triangle trigonometry, but not unit-circle trigonometry. This causes problems with complex numbers.

Classroom implementation:

- Implication: Use the circle simulation.

Complex Plane

$\mathbb{C} = \mathbb{R} \oplus i\mathbb{R}$



$i^2 = -1$

$$(x, y) \longmapsto x + iy$$

$$x + iy = r\cos\theta + i\,r\sin\theta = r\,e^{i\theta}$$

Special case: $e^{\pm i\pi/2} = \pm i$

$$e^{i\pi} + 1 = 0$$

Representing Complex Numbers

- Please stand up.
- Use left hand.
- Real axis points forward.
- Imaginary axis points upward.

Show me:

- 1
- $2\,i$
- $1 + i$
- $e^{-i\pi/3}$

Kinesthetic Activity

Things to consider:

- Everyone is awake!
- Teacher can see what everyone is thinking.
- Highlights geometric reasoning.
- Students get geometric cues from others.
- Students must make a decision.
- Student can be asked to translate representations.

Classroom implementation:

- *Please stand up.*
- *Show me...*
- *Thank you, you can sit down.*

Multiplication by i

$$\boxed{(1+i)i = i - 1}$$

If $1+i$ is multiplied by i, the corresponding vector is:

1: Reflected about the x-axis

2: Reflected about the y-axis

3: Rotated by $\frac{\pi}{2}$ (90°) counterclockwise

4: Rotated by $\frac{\pi}{2}$ (90°) clockwise

DO NOT VOTE UNTIL TOLD TO DO SO!

Multiplication by i

$$(1 + 2i)i$$

If $1 + 2i$ is multiplied by i, the corresponding vector is:

1: Reflected about the x-axis

2: Reflected about the y-axis

3: Rotated by $\frac{\pi}{2}$ (90°) counterclockwise

4: Rotated by $\frac{\pi}{2}$ (90°) clockwise

DO NOT VOTE UNTIL TOLD TO DO SO!

Multiplication by i

$$(re^{i\theta})i$$

If $re^{i\theta}$ is multiplied by i, the corresponding vector is

1: Reflected about the x-axis

2: Reflected about the y-axis

3: Rotated by $\frac{\pi}{2}$ (90°) counterclockwise

4: Rotated by $\frac{\pi}{2}$ (90°) clockwise

DO NOT VOTE UNTIL TOLD TO DO SO!

Things to consider:

- Asks students to make a commitment.
- Asks students to defend an answer.
- Good questions: conceptual, focus on common mistakes.

Classroom implementation:

- Many "response" systems:
  clickers, ABCD cards, whiteboards, fingers.
- Two stages.
- Simultaneous and anonymous.
- *Convince your neighbor.*

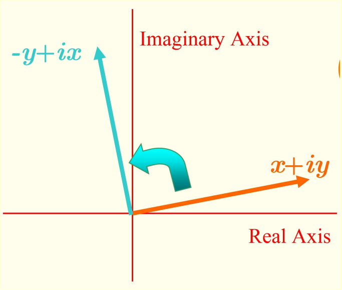
Multiplication by $se^{i\alpha}$

$$(re^{i\theta})(se^{i\alpha})$$

If $re^{i\theta}$ is multiplied by $se^{i\alpha}$, the corresponding vector is...

WRITE YOUR ANSWER ON YOUR SMALL WHITE BOARD!

Multiplication by i



Multiplication by i: $(x + i\,y)i = i\,x + i^2 y = -y + ix$

Rotates counterclockwise by $\pi/2$

Multiplication by $s\,e^{i\alpha}$: $(r\,e^{i\theta})(s\,e^{i\alpha}) = rs\,e^{i(\theta+\alpha)}$

Rotates counterclockwise by α
and stretches by s

Sequences of Questions

Things to consider:

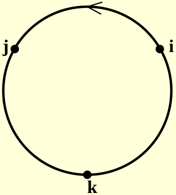
- Frame the sequence with increasing sophistication.
- Choose clicker questions vs. SWBQs by need for open-endedness.
- Choose clicker questions vs. SWBQs by type of response desired.

Classroom implementation:

- SWBQs: Gather responses and discuss.
- Use wrap-up as an opportunity for reflection.
- SWBQs can be spontaneous.

Quaternions

$\mathbb{H} = \mathbb{C} \oplus \mathbb{C}j$



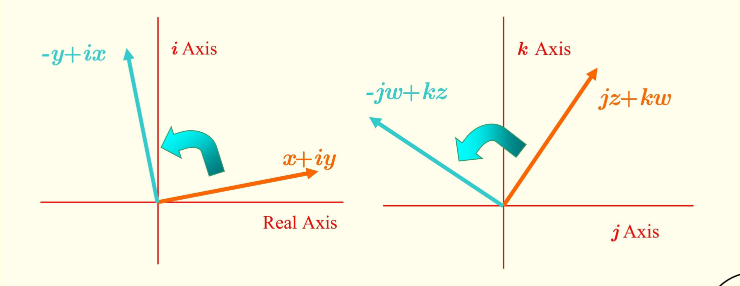
$$q = (x + y\,i) + (z + w\,i)\,j = x + y\,i + z\,j + w\,k$$

$$\boxed{ij = k = -ji;\, i^2 = j^2 = k^2 = -1}$$

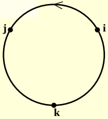
$\mathbb{H}$ is for Hamilton! ($\mathbb{Q}$ denotes rationals)

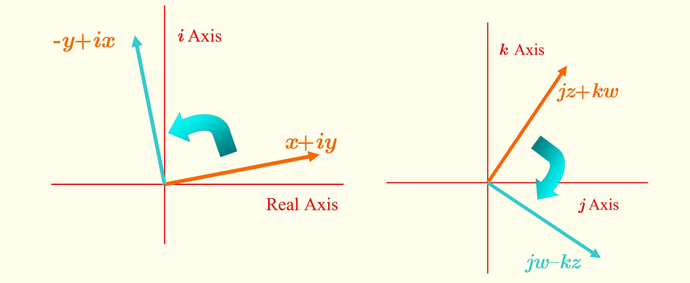
Calculate with your group: iq and qi

iq vs. *qi*



$$q = x + iy + jz + kw$$
$$iq = ix - y + kz - jw$$
$$qi = ix - y - kz + jw$$





Small Group Activity

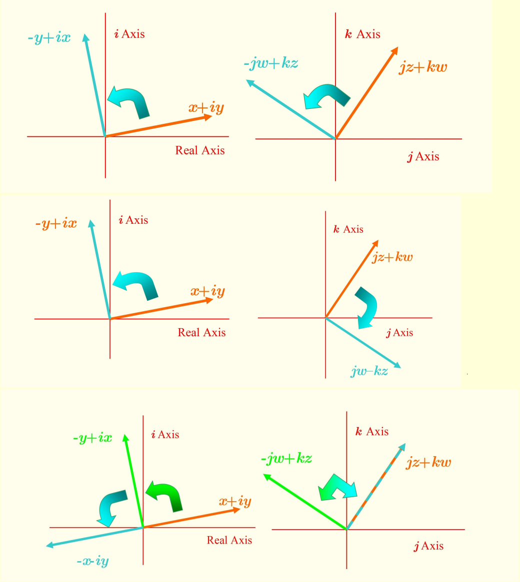
Things to consider:

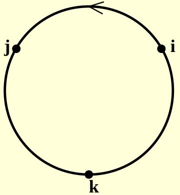
- Can emphasize more complex problems/reasoning.
- Students practice problem solving themselves.
- Equity: moves office hours into the classroom.

Classroom implementation:

- *You have 10 minutes; GO!*
- Who needs help?
- Do you need more time?
- Pause.

Conjugation





$$q = x + iy + jz + kw$$

$$iq = ix - y + kz - jw$$
$$qi = ix - y - kz + jw$$

$$iqi = -x - iy + jz + kw$$
$$-iqi = x + iy - jz - kw$$
(rotation in jk-plane)

Lectures/Slides/Figures

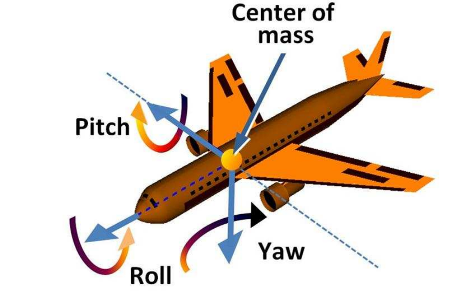
Things to consider:

- Lecture is fast; use it when it works.
- What is the focus of attention? (You, the slides, their notes...)
- How busy are the slides?
- Do the figures have distracting elements?

Classroom implementation:

- Have a way to show students where you are on the slide.

Rotations



$$q \longmapsto e^{i\theta/2} q e^{-i\theta/2}$$

- $1 \longmapsto 1$; $i \longmapsto i$
- Rotates by θ "about i" (in jk-plane)
- $q \longmapsto e^{j\theta/2} q e^{-j\theta/2}$ rotates about j, etc.

$\therefore$ SO(3), the rigid rotations in 3 dimensions

▸ Rotations

Simulation/Demo

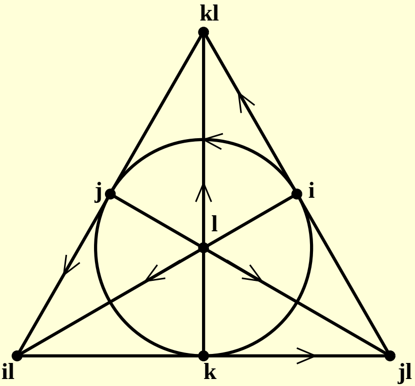
Things to consider:

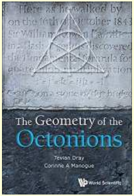
- Decide between black box or open coding.
- Show geometry and/or time dependence.
- Plan specific questions: Students need to be taught to ask relevant questions or to explore parameter space.

Classroom implementation:

- Stand behind students to see if they are having problems with the computer.

Generalizations





2015

Octonions! $(\mathbb{O} = \mathbb{H} + \mathbb{H}\ell)$

Use to model particle physics

http://octonions.geometryof.org/GO

Plum Muffins

Story telling is memorable.

Things to consider:

- How much detail should you include?
- How closely should the notes follow the course?

Classroom implementation:

- How do you counter the ideas: "I have the notes on my computer, so I understand the content" or "I have the notes on my computer, so I will study them later"?
- When should students have access to the notes?

Lecture (vs. Activities)

The Instructor:

- Paints big picture
- Inspires.
- Covers lots fast.
- Models speaking.
- Models problem-solving.
- Controls questions.
- Makes connections.
- Demonstrates new complicated reasoning.

The Students:

- Focus on subtleties.
- Experience delight.
- Slow, but in depth.
- Practice speaking.
- Practice problem-solving.
- Control questions.
- Make connections.
- Discover questions about what is complicated.

Please Return:

Please clean up your toys:

- Erase your whiteboard.
- Return the whiteboard and marker.