

Active Engagement: Lessons learned from the Paradigms and Bridge projects

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Get Ready:

Please sit with one or two other partners.

For each group, please pick up:

- 1 small whiteboard;
- 1 whiteboard pen;
- 1 high-tech eraser (e.g. tissue/napkin);

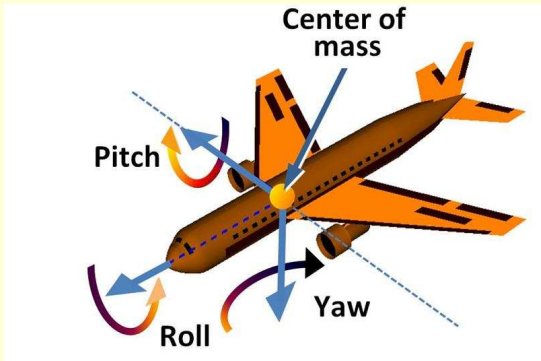
Good teaching is like picking up someone else's baby.

Using the Quaternions to Implement Rotations

Tevian Dray & Corinne Manogue

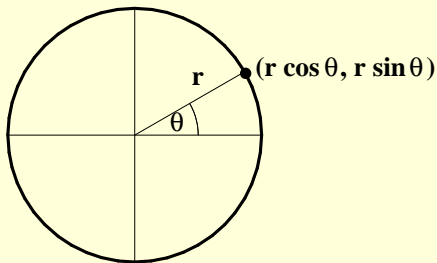
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3-d rotations: Aeronautics, robotics, computer graphics, ...

New Content: Use quaternions to implement rotations



Polar coordinates: $x = r \cos \theta; y = r \sin \theta.$

► Circle Trig

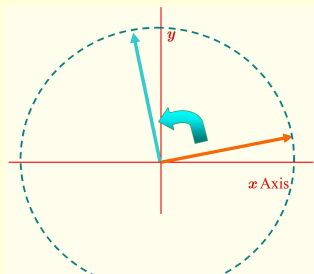
Things to consider:

- Whenever possible, base your instruction on what is known about incoming student resources.
- Example: Dr. Emily Smith (OSU 2016) showed that many upper-division physics students know triangle trigonometry, but not unit-circle trigonometry. This causes problems with complex numbers.

Classroom implementation:

- Implication: Use the circle simulation.

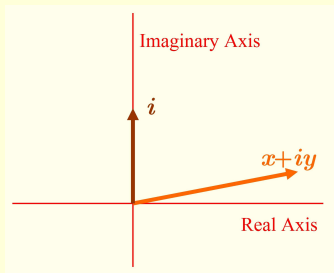
Rotations



▶ Rotations

Complex Plane

$$\mathbb{C} = \mathbb{R} \oplus i\mathbb{R}$$



$$i^2 = -1$$

$$(x, y) \mapsto x + iy$$

$$x + iy = r \cos \theta + i r \sin \theta = r e^{i\theta}$$

$$\text{Special case: } e^{\pm i\pi/2} = \pm i$$

$$e^{i\pi} + 1 = 0$$

Representing Complex Numbers

- Please stand up.
- Use left hand.
- Real axis points forward.
- Imaginary axis points upward.

Show me:

- 1
- $2i$
- $1 + i$
- $e^{-i\pi/3}$

Things to consider:

- Everyone is awake!
- Teacher can see what everyone is thinking.
- Highlights geometric reasoning.
- Students get geometric cues from others.
- Students must make a decision.
- Student can be asked to translate representations.

Classroom implementation:

- *Please stand up.*
- *Show me...*
- *Thank you, you can sit down.*

$$(1 + i)i = i - 1$$

If $1 + i$ is multiplied by i , the corresponding vector is:

- 1: Reflected about the x -axis
- 2: Reflected about the y -axis
- 3: Rotated by $\frac{\pi}{2}$ (90°) counterclockwise
- 4: Rotated by $\frac{\pi}{2}$ (90°) clockwise

DO NOT VOTE UNTIL TOLD TO DO SO!

Multiplication by i

$$(1 + 2i)i$$

If $1 + 2i$ is multiplied by i , the corresponding vector is:

- 1: Reflected about the x -axis
- 2: Reflected about the y -axis
- 3: Rotated by $\frac{\pi}{2}$ (90°) counterclockwise
- 4: Rotated by $\frac{\pi}{2}$ (90°) clockwise

DO NOT VOTE UNTIL TOLD TO DO SO!

Multiplication by i

$$(re^{i\theta})i$$

If $re^{i\theta}$ is multiplied by i , the corresponding vector is

- 1:** Reflected about the x -axis
- 2:** Reflected about the y -axis
- 3:** Rotated by $\frac{\pi}{2}$ (90°) counterclockwise
- 4:** Rotated by $\frac{\pi}{2}$ (90°) clockwise

DO NOT VOTE UNTIL TOLD TO DO SO!

Things to consider:

- Asks students to make a commitment.
- Asks students to defend an answer.
- Good questions: conceptual, focus on common mistakes.

Classroom implementation:

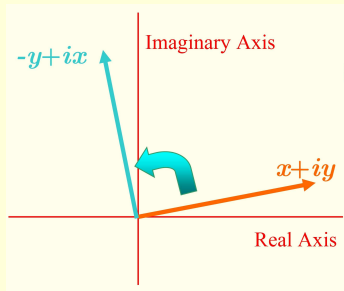
- Three “response” systems: clickers, ABCD cards, whiteboards.
- Two stages.
- Simultaneous and anonymous.
- *Convince your neighbor.*

$$(re^{i\theta})(se^{i\alpha})$$

If $re^{i\theta}$ is multiplied by $se^{i\alpha}$, the corresponding vector is...

WRITE YOUR ANSWER ON YOUR SMALL WHITE BOARD!

Multiplication by i



Multiplication by i : $(x + iy)i = ix + i^2y = -y + ix$

Rotates counterclockwise by $\pi/2$

Multiplication by $s e^{i\alpha}$: $(r e^{i\theta})(s e^{i\alpha}) = rs e^{i(\theta+\alpha)}$

Rotates counterclockwise by α
and stretches by s

Things to consider:

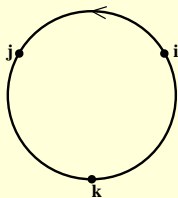
- Frame the sequence with increasing sophistication.
- Choose clicker questions vs. SWBQs by need for open-endedness.
- Choose clicker questions vs. SWBQs by type of response desired.

Classroom implementation:

- SWBQs: Gather responses and discuss.
- Use wrap-up as an opportunity for reflection.
- SWBQs can be spontaneous.

Quaternions

$$\mathbb{H} = \mathbb{C} \oplus \mathbb{C}j$$



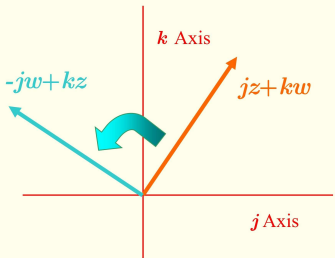
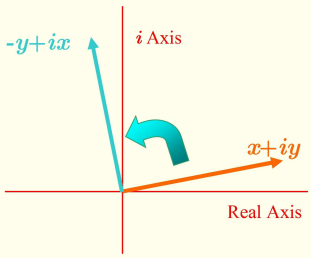
$$q = (x + yi) + (z + wi)j = x + yi + zj + wk$$

$$ij = k = -ji; i^2 = j^2 = k^2 = -1$$

\mathbb{H} is for Hamilton! (\mathbb{Q} denotes rationals)

Calculate with your group: iq and qi

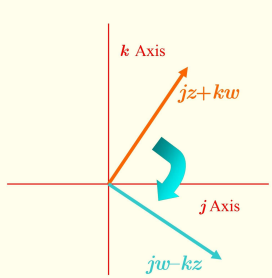
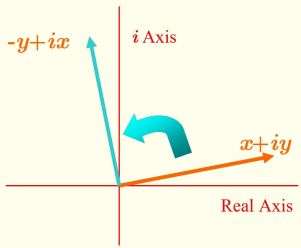
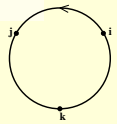
iq vs. qi



$$q = x + iy + jz + kw$$

$$iq = ix - y + kz - jw$$

$$qi = ix - y - kz + jw$$



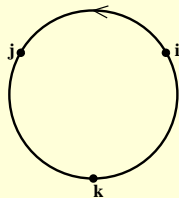
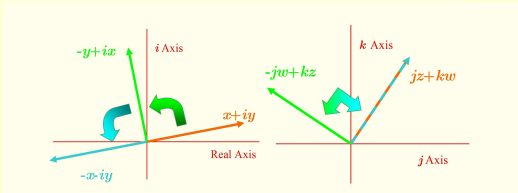
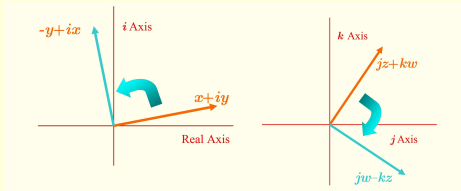
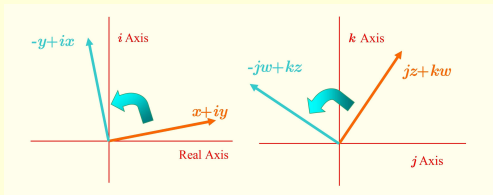
Things to consider:

- Can emphasize more complex problems/reasoning.
- Students practice problem solving themselves.
- Equity: moves office hours into the classroom.

Classroom implementation:

- *You have 10 minutes; GO!*
- Who needs help?
- Do you need more time?
- Pause.

Conjugation



$$q = x + iy + jz + kw$$

$$iq = ix - y + kz - jw$$

$$qi = ix - y - kz + jw$$

$$iqi = -x - iy + jz + kw$$

$$-iqi = x + iy - jz - kw$$

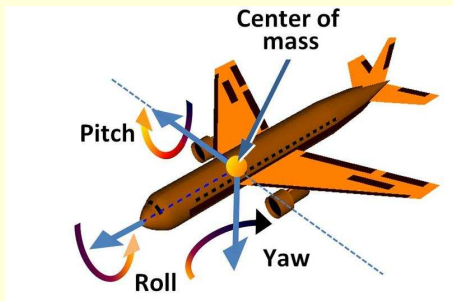
(rotation in jk -plane)

Things to consider:

- Lecture is fast; use it when it works.
- What is the focus of attention? (You, the slides, their notes...)
- How busy are the slides?
- Do the figures have distracting elements?

Classroom implementation:

- Have a way to show students where you are on the slide.



$$q \mapsto e^{i\theta/2} q e^{-i\theta/2}$$

- $1 \mapsto 1; i \mapsto i$
- Rotates by θ "about i " (in jk -plane)
- $q \mapsto e^{j\theta/2} q e^{-j\theta/2}$ rotates about j , etc.

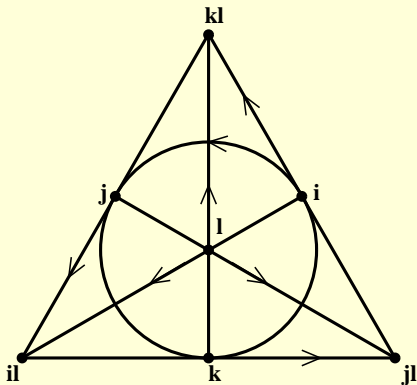
\therefore SO(3), the rigid rotations in 3 dimensions

Things to consider:

- Decide between black box or open coding.
- Show geometry and/or time dependence.
- Plan specific questions: Students need to be taught to ask relevant questions or to explore parameter space.

Classroom implementation:

- Stand behind students to see if they are having problems with the computer.



Octonions! ($\mathbb{O} = \mathbb{H} + \mathbb{H}\ell$)

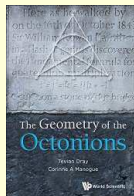
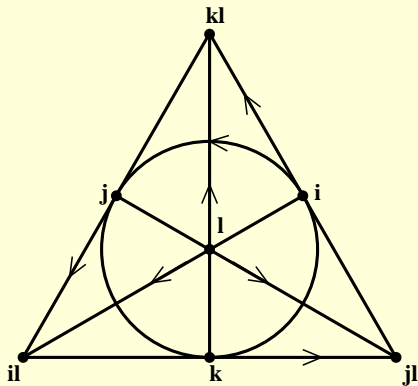
Use to model particle physics

<http://octonions.geometryof.org/G0>

Plum Muffins

Story telling is memorable.

Generalizations



2015

Octonions! ($\mathbb{O} = \mathbb{H} + \mathbb{H}\ell$)

Use to model particle physics

<http://octonions.geometryof.org/G0>

Things to consider:

- How much detail should you include?
- How closely should the notes follow the course?

Classroom implementation:

- How do you counter the ideas: “I have the notes on my computer, so I understand the content” or “I have the notes on my computer, so I will study them later”?
- When should students have access to the notes?

The Instructor:

- Paints big picture
- Inspires.
- Covers lots fast.
- Models speaking.
- Models problem-solving.
- Controls questions.
- Makes connections.
- Demonstrates new complicated reasoning.

The Students:

- Focus on subtleties.
- Experience delight.
- Slow, but in depth.
- Practice speaking.
- Practice problem-solving.
- Control questions.
- Make connections.
- Discover questions about what is complicated.

Please Return:

Please clean up your toys:

- Erase your whiteboard.
- Return the whiteboard and marker.