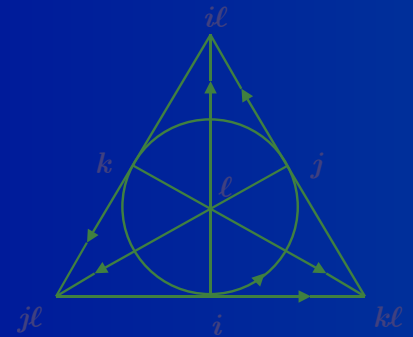
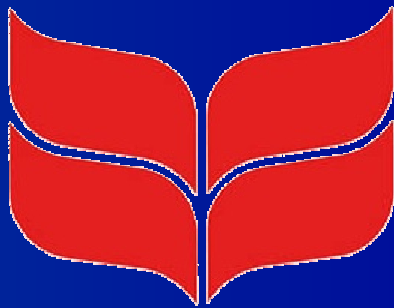


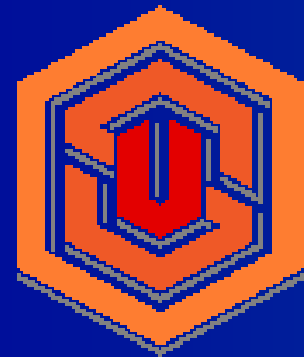
What Does Geometry Tell Us About the Universe?



Tevian Dray & Corinne Manogue



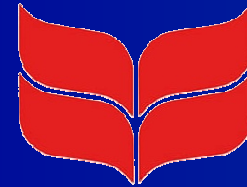
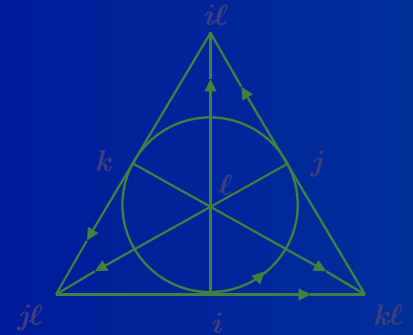
GRINNELL COLLEGE



Oregon State University

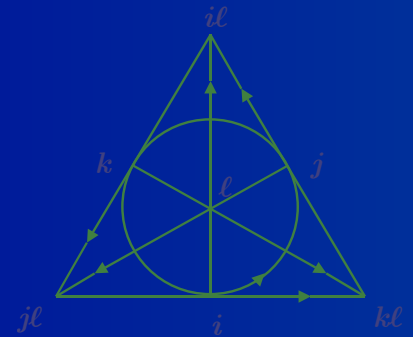
Support

- Grinnell College
 - Noyce Visiting Professorship
- Mount Holyoke College
 - Hutchcroft Fund
- Oregon State University
 - Department of Mathematics
 - Department of Physics
- National Science Foundation
 - DUE-9653250
 - DUE-0088901

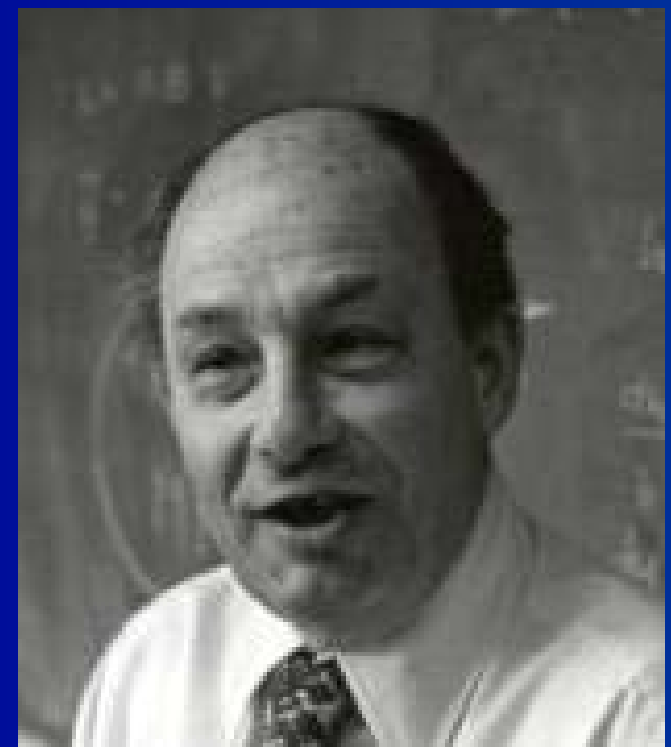


Peter Bergmann

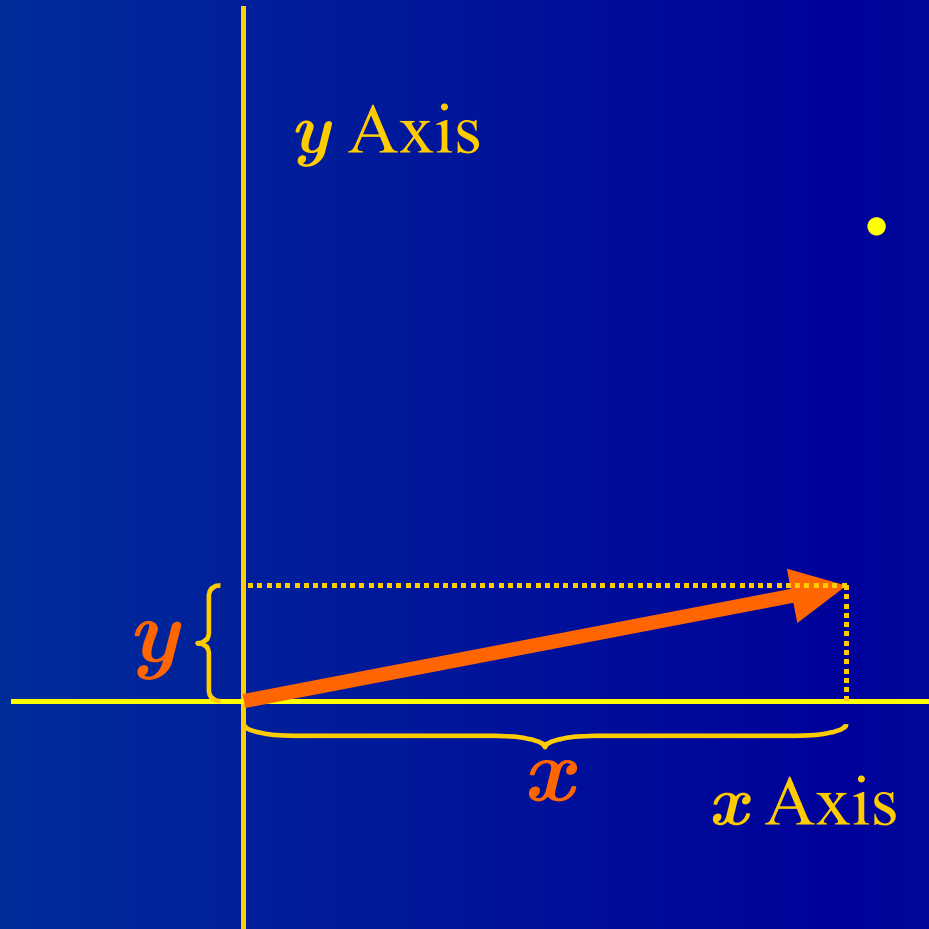
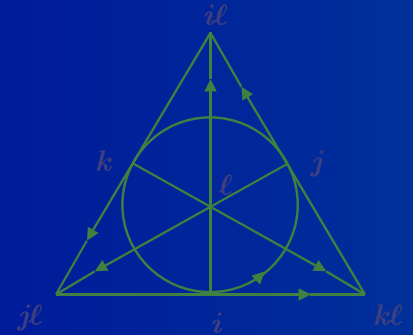
March 24, 1915—October 19, 2002



- Friend and Mentor
- Worked with Einstein
- First text on general relativity.
- Einstein Prize: *"For pioneering investigations in general relativity, ... and for leadership and inspiration to generations of researchers in general relativity."*



Vectors

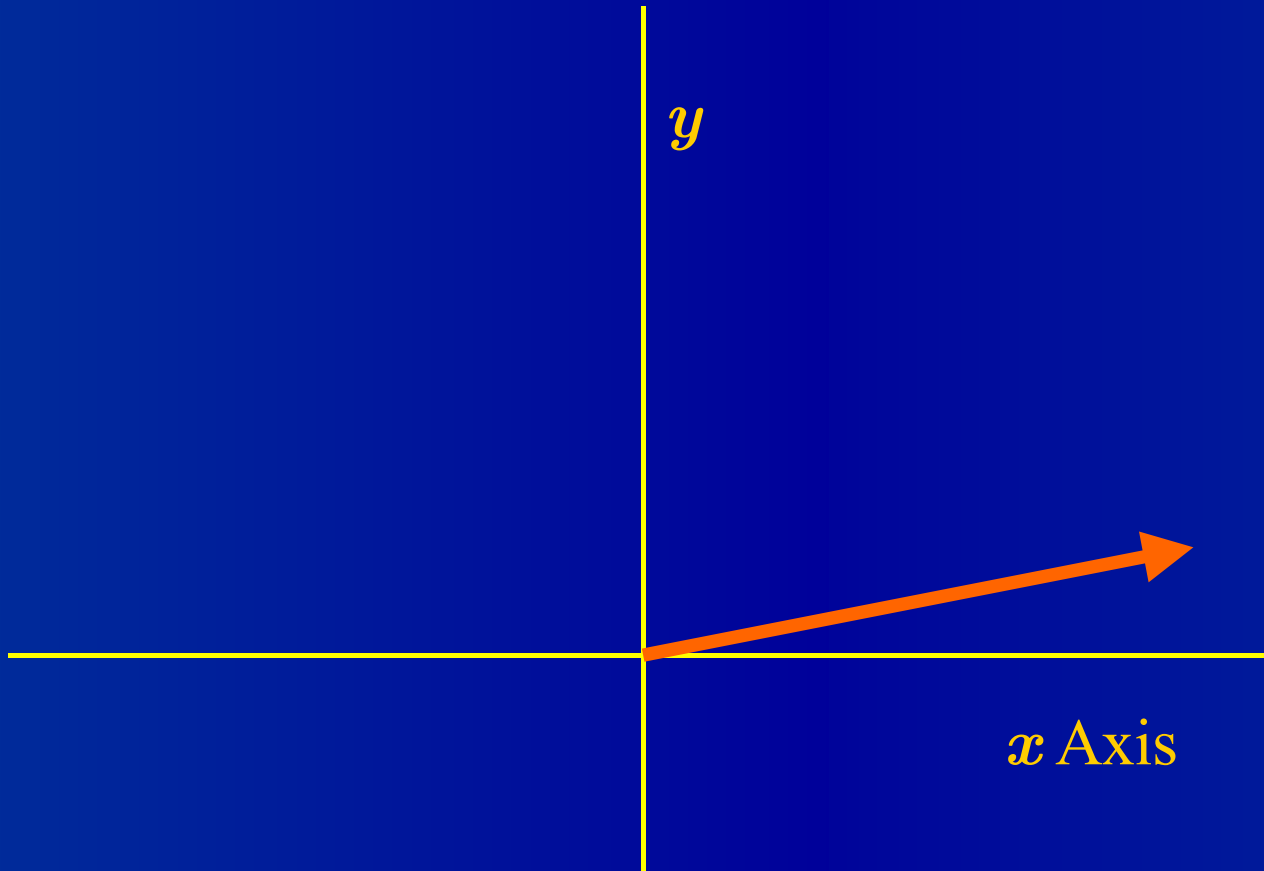
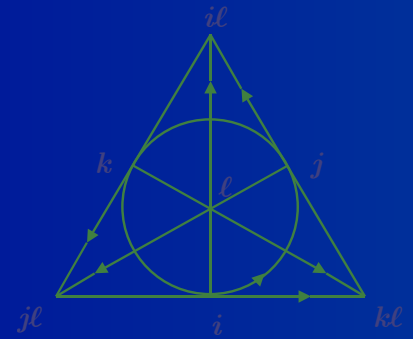


- Vectors have length and direction

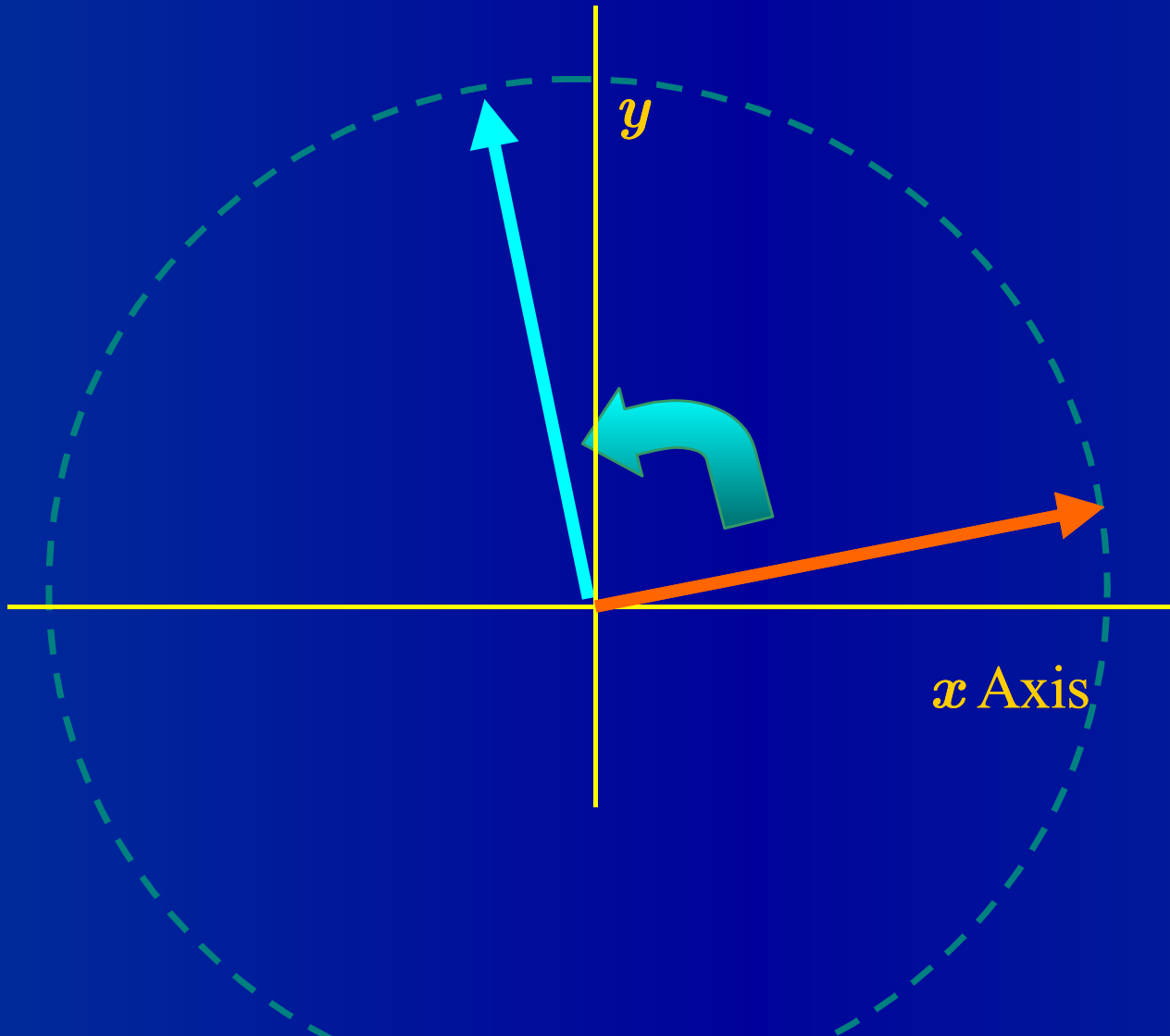
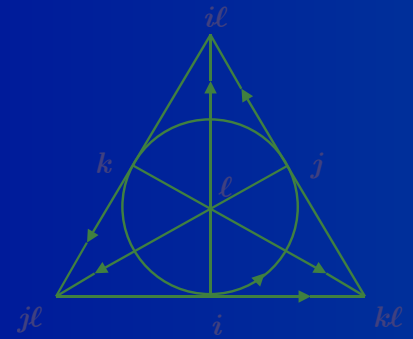
$$\text{length}^2 = x^2 + y^2$$

(Pythagorean Theorem)

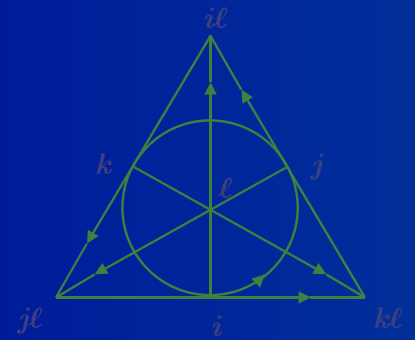
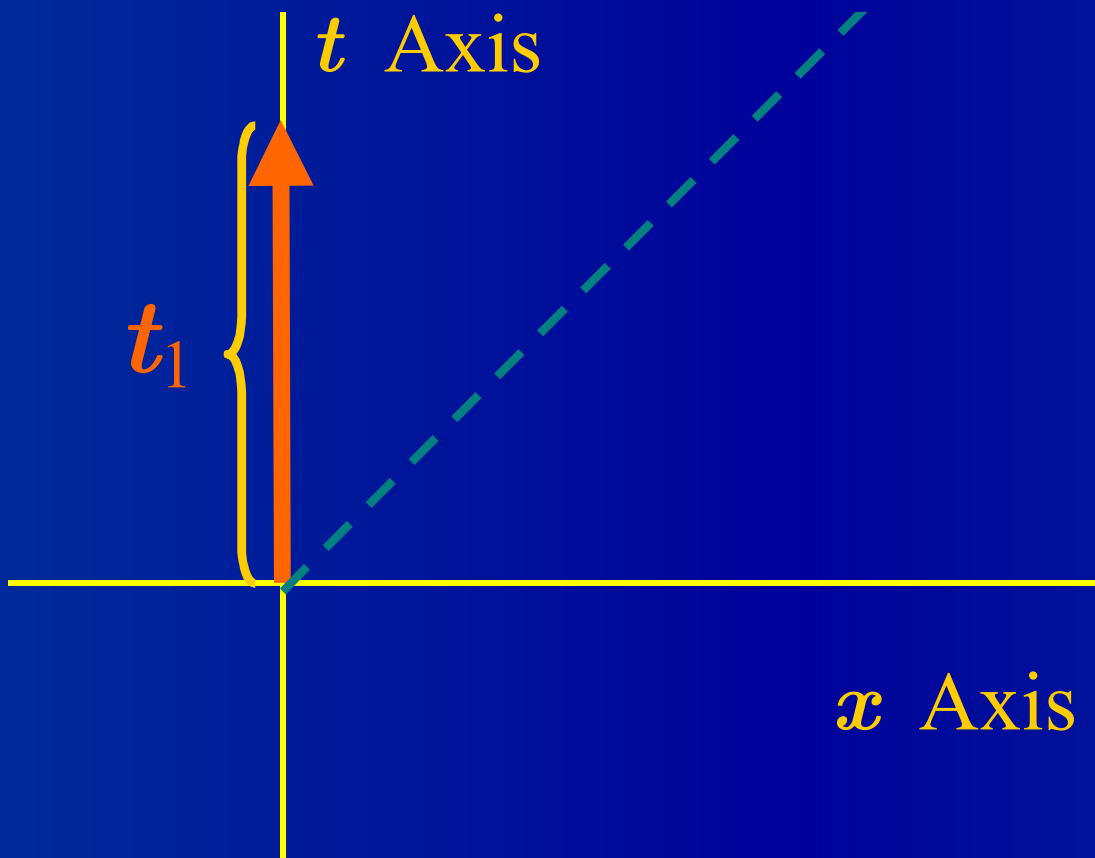
Vector Rotations



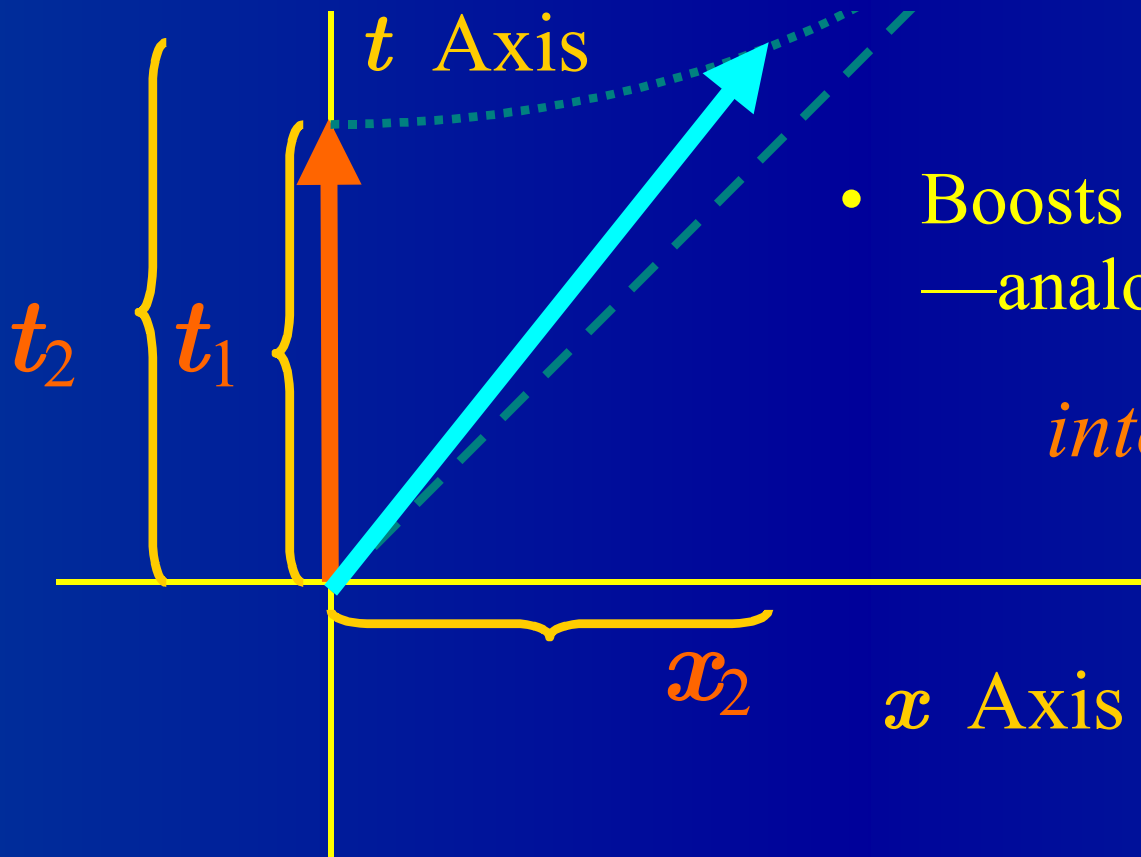
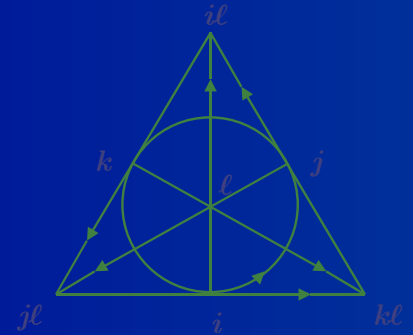
Vector Rotations



Spacetime



Spacetime



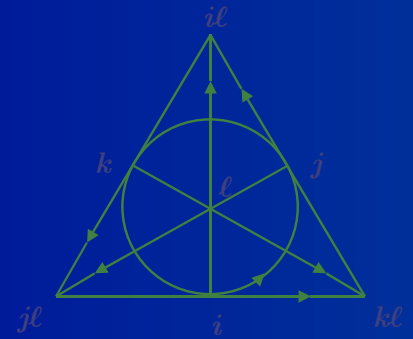
- Boosts preserve “interval,”
—analogous to length.

$$interval^2 = x^2 - t^2$$

Tevatron

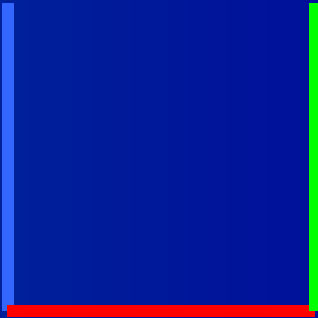
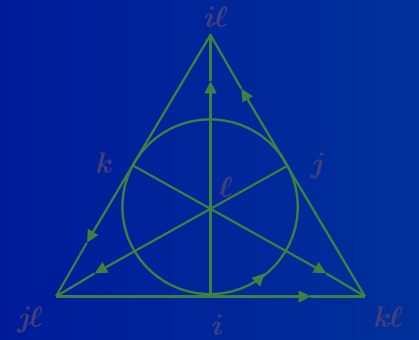


Fermilab Photo

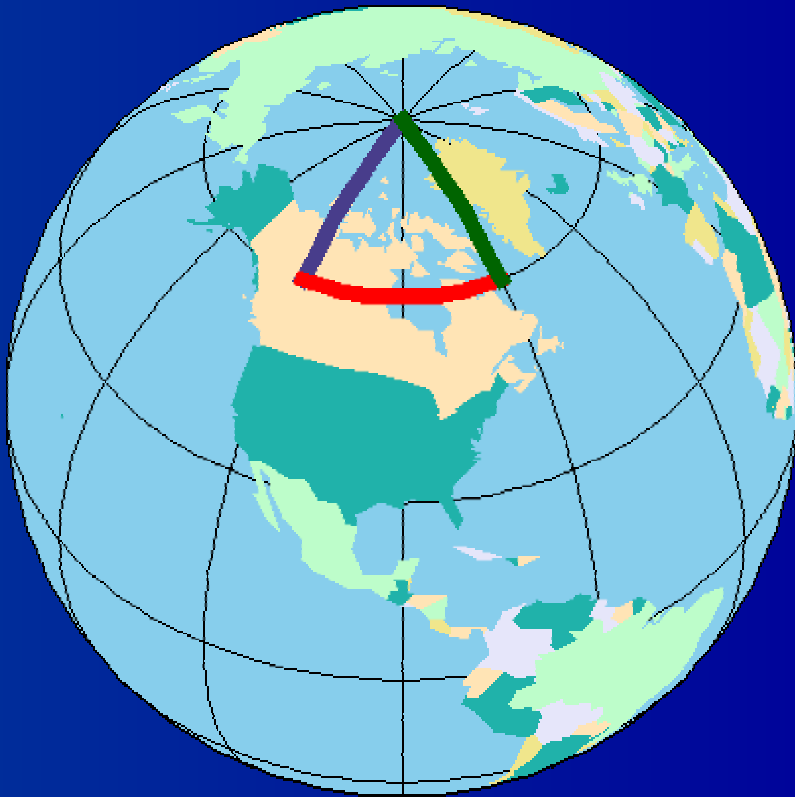
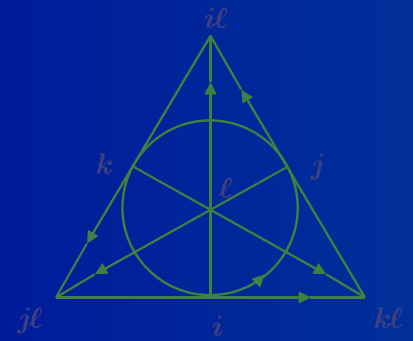


- Particle accelerators confirm “interval invariance” every day.

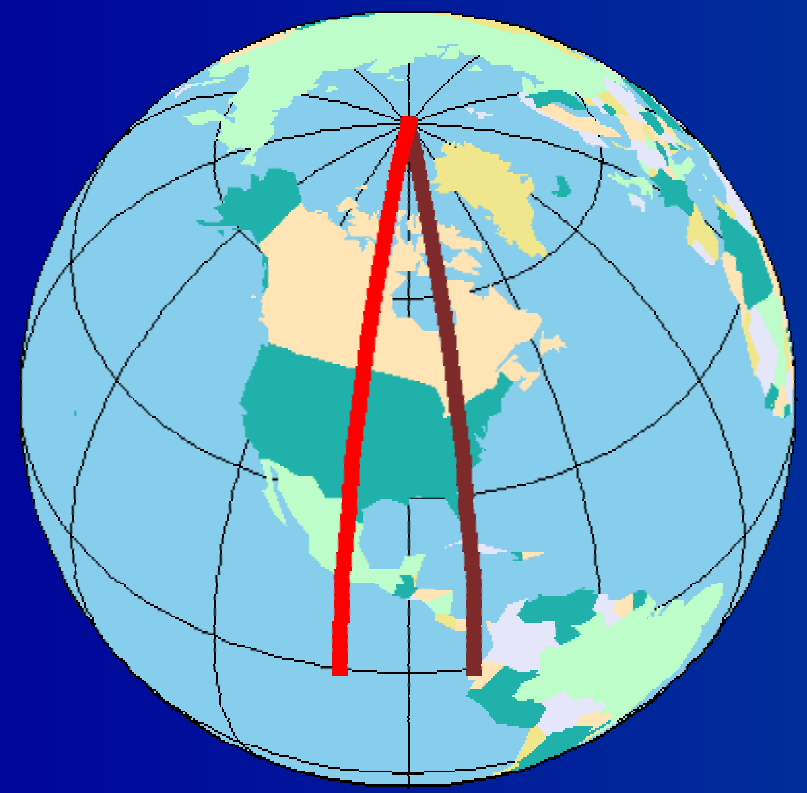
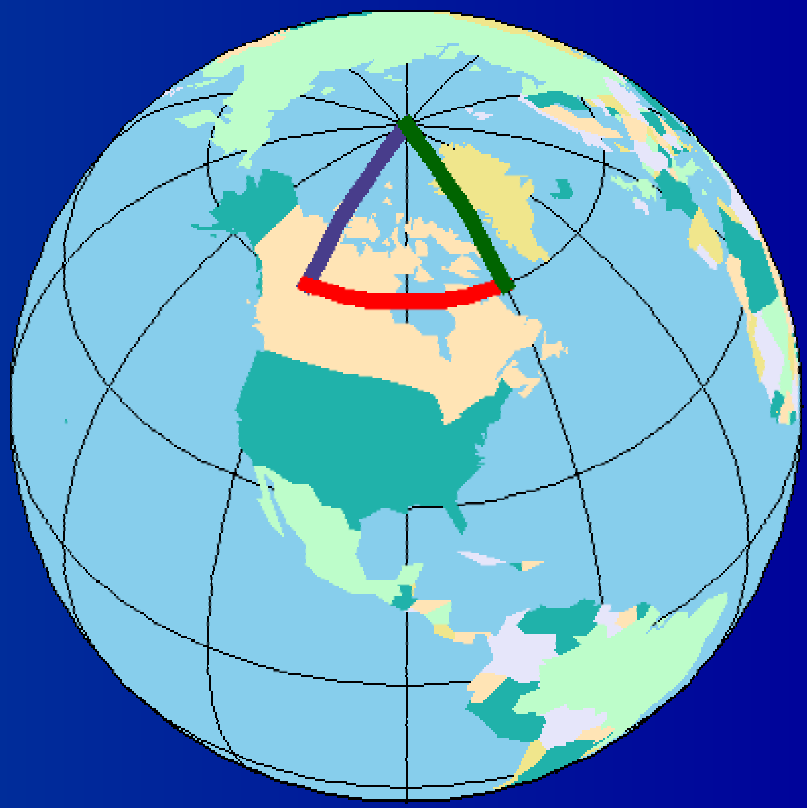
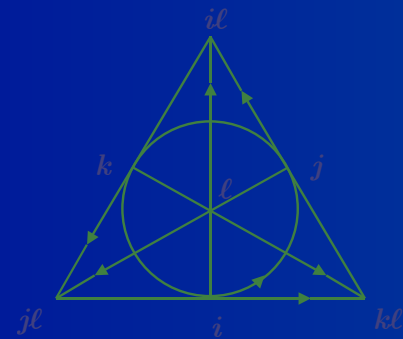
Explorers



Explorers



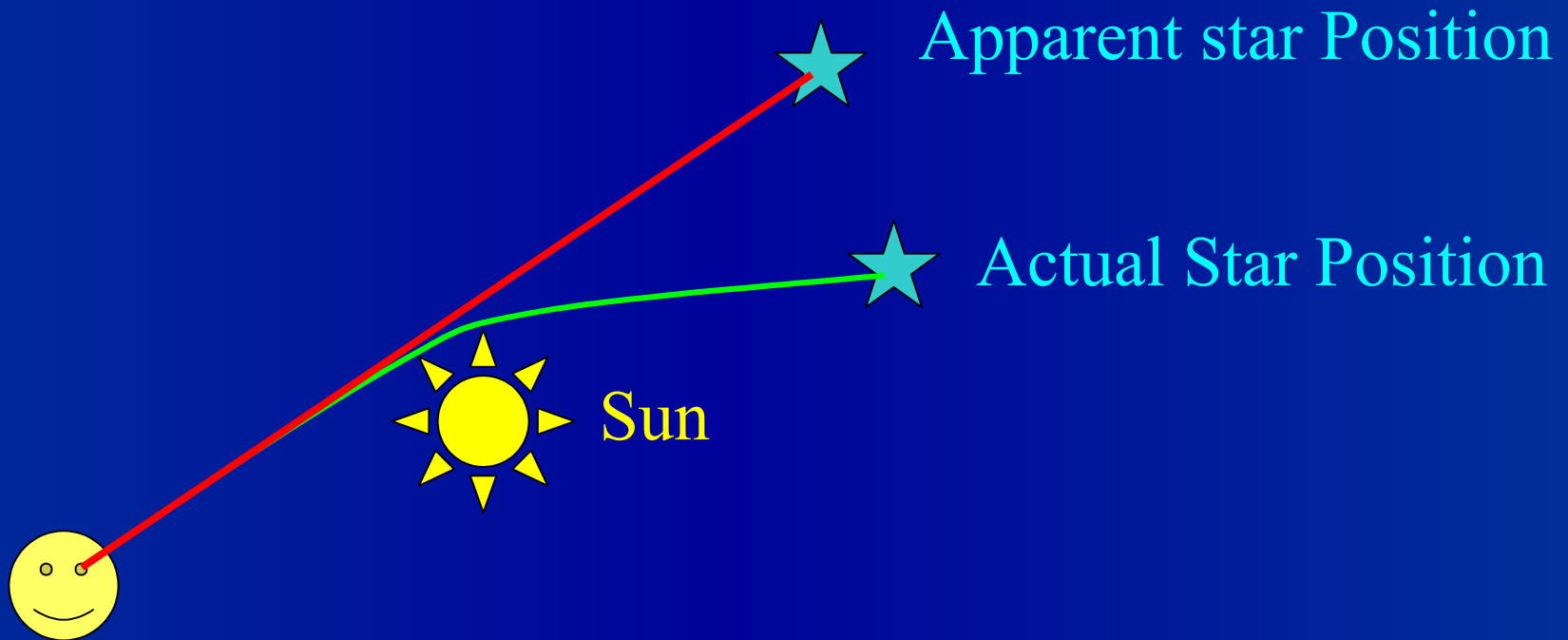
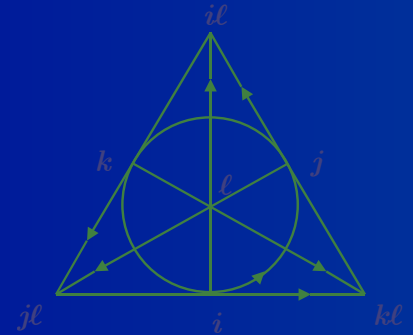
Explorers



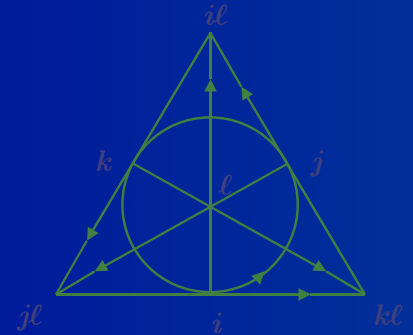
Spacetime Curvature

General Relativity—

mass produces curvature of spacetime.



Curved Spacetime

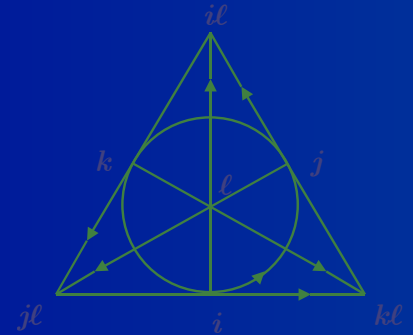
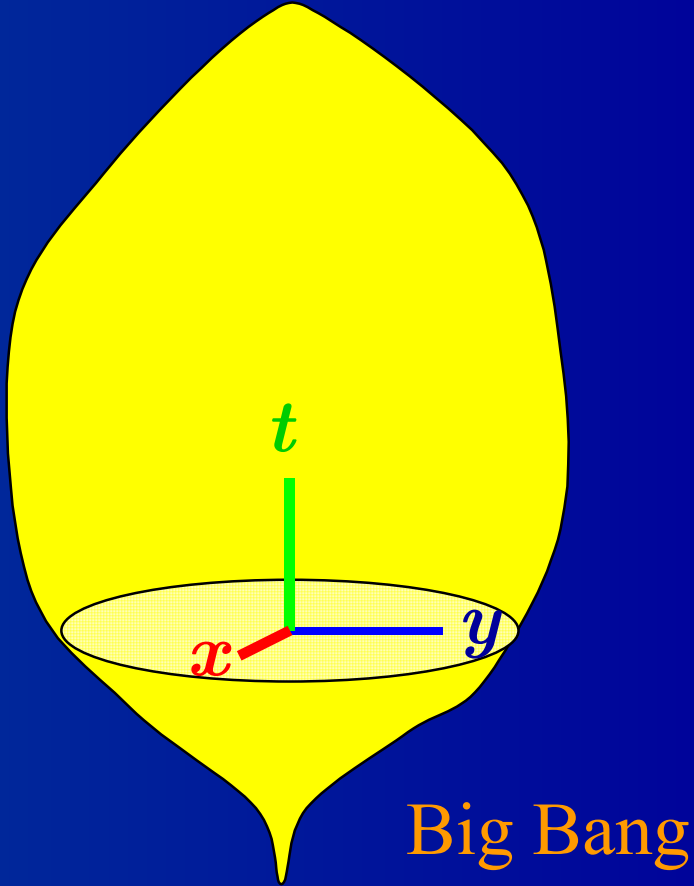


- A cluster of galaxies in the foreground acts as a lens for galaxies 5-10 times more distant.

Photo from public archives of the Space Science Telescope Institute (<http://www.stsci.edu/public.html>)

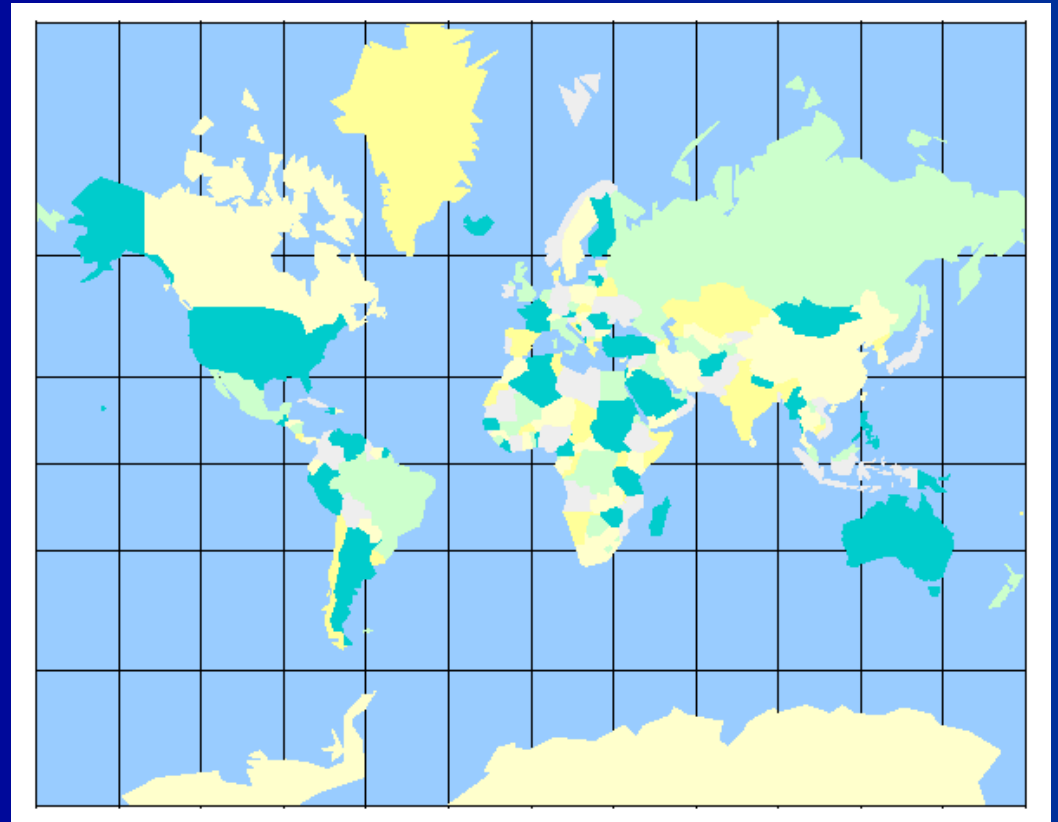
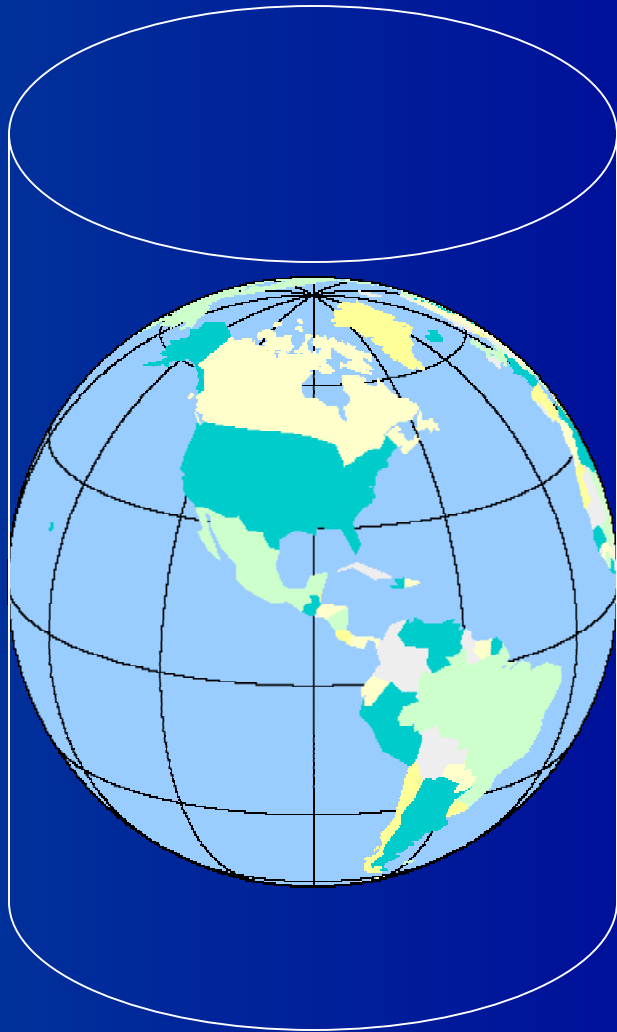
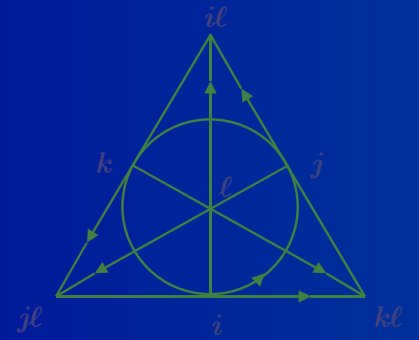
Big Bang

Big Crunch?

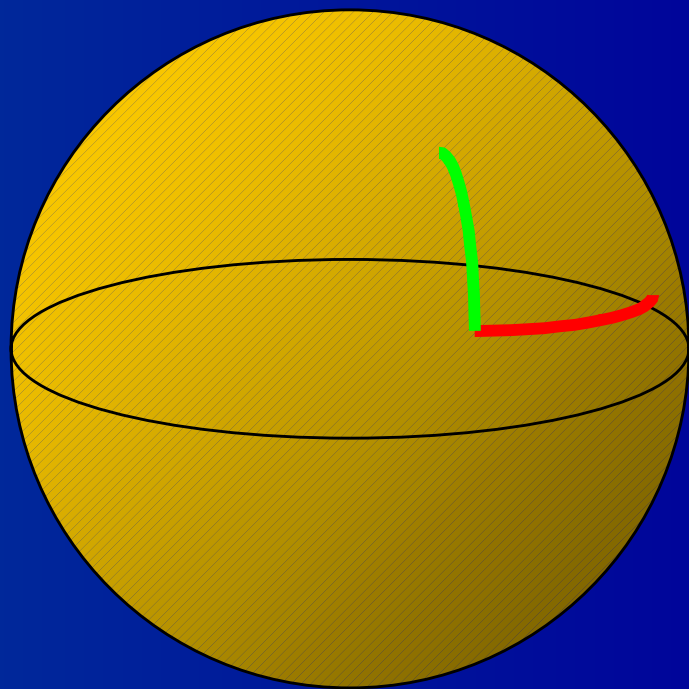
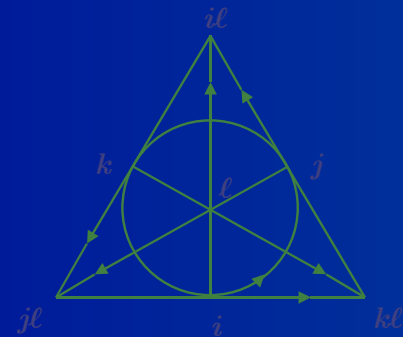


- Space & Time are related.
- Spacetime is curved.
- Galaxies are moving apart.
- ➔ Space and time are part of the universe.

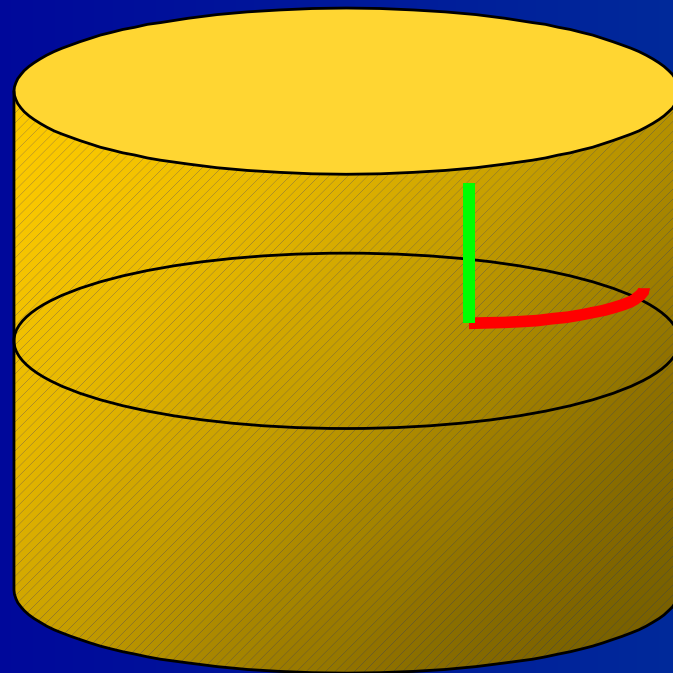
Curvature



Symmetries



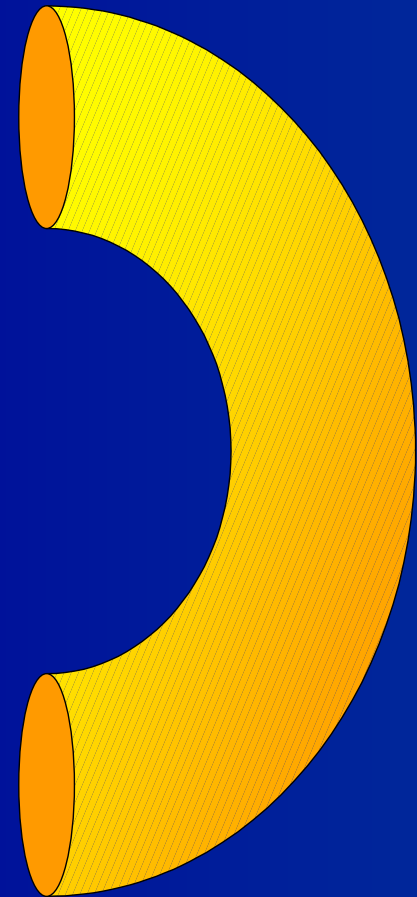
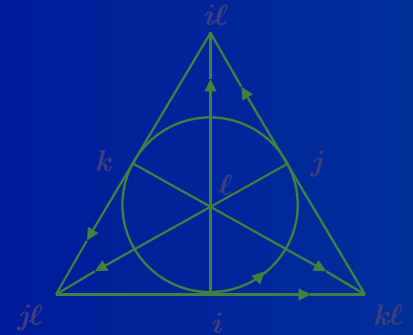
SU(2)



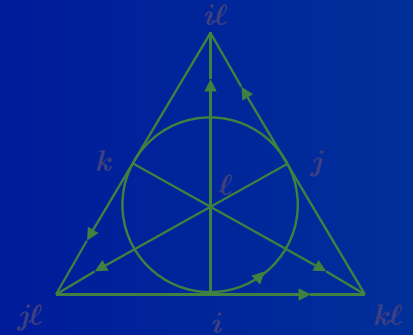
U(1)

Superstrings

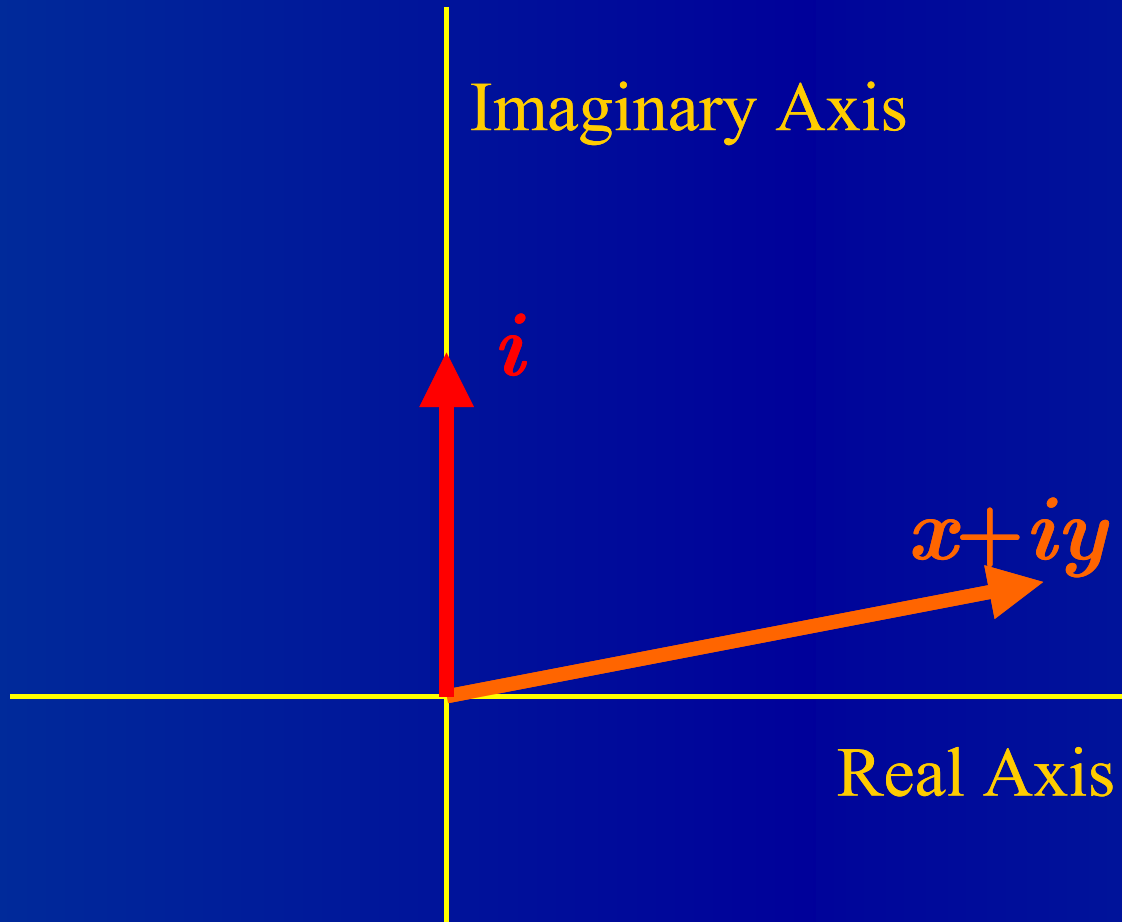
- Superstrings live in 10 spacetime dimensions.
- 6 dimensions are curled up in a Calabi-Yau manifold.
- Symmetries determine the types of particles.



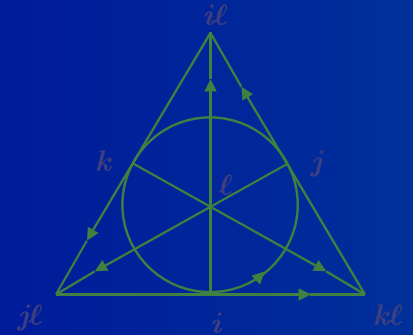
Complex Numbers



$$i^2 = -1$$



Multiplication by i



Imaginary Axis

$$(x + iy)i$$

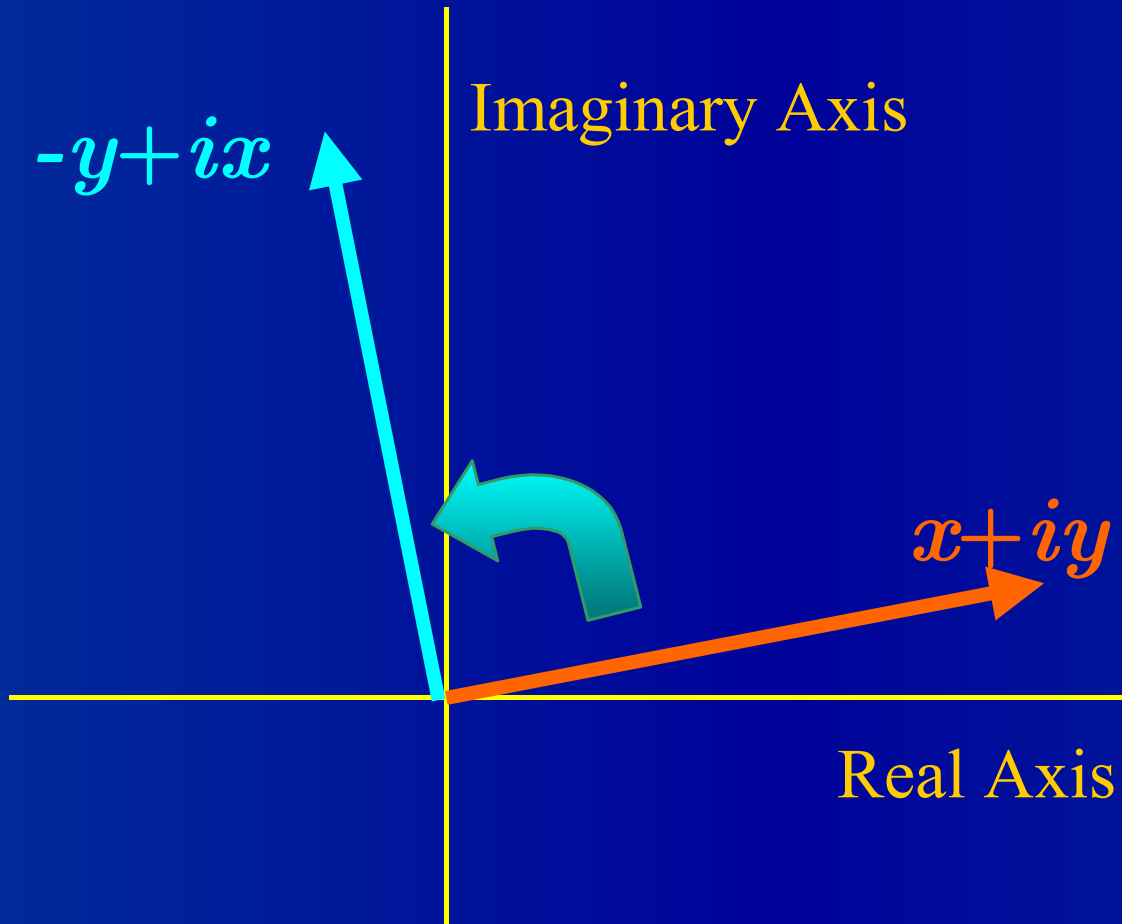
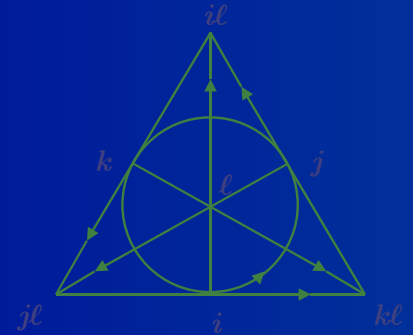
$$= ix + (i)^2 y$$

$$= -y + ix$$

$x + iy$

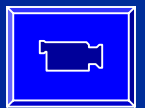
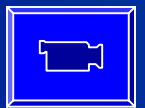
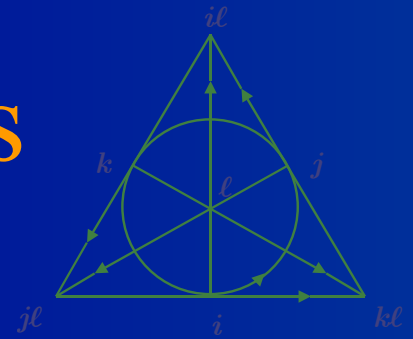
Real Axis

Multiplication by i

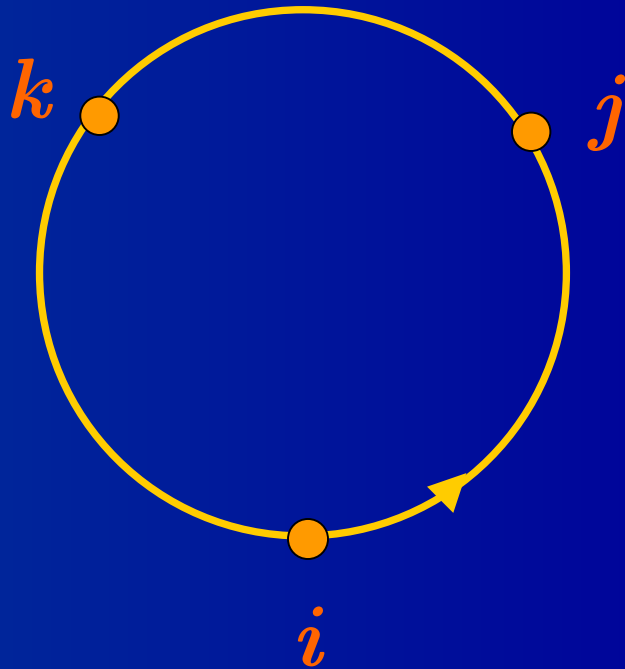
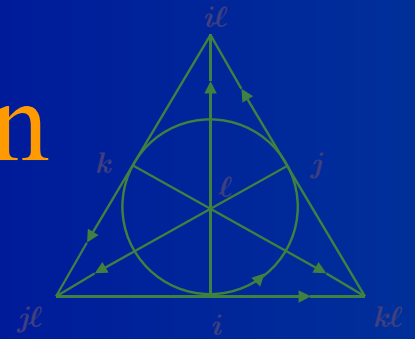


$$\begin{aligned}(x + iy)i &= ix + (i)^2 y \\ &= -y + ix\end{aligned}$$

Order of Rotations Matters



Quaternionic Multiplication

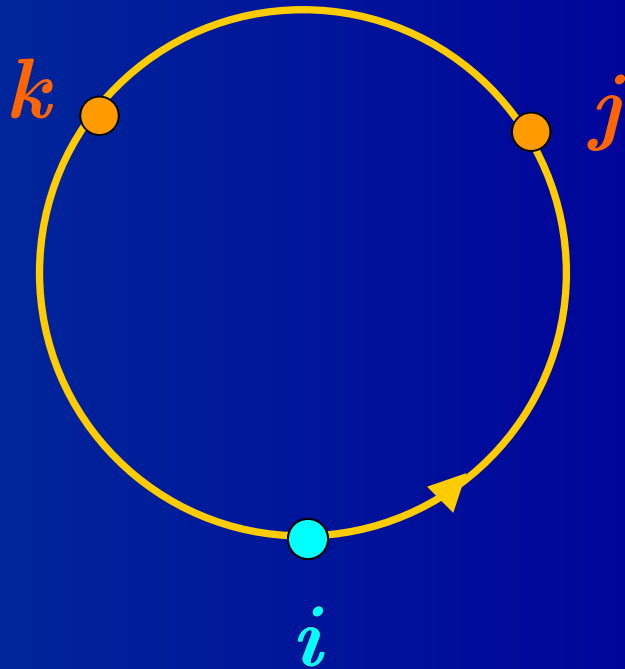
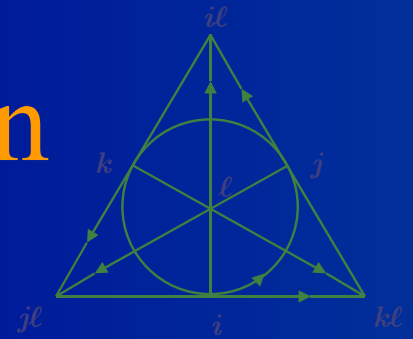


$$i^2 = j^2 = k^2 = -1$$

$$i j = k$$

$$j i = -k$$

Quaternionic Multiplication

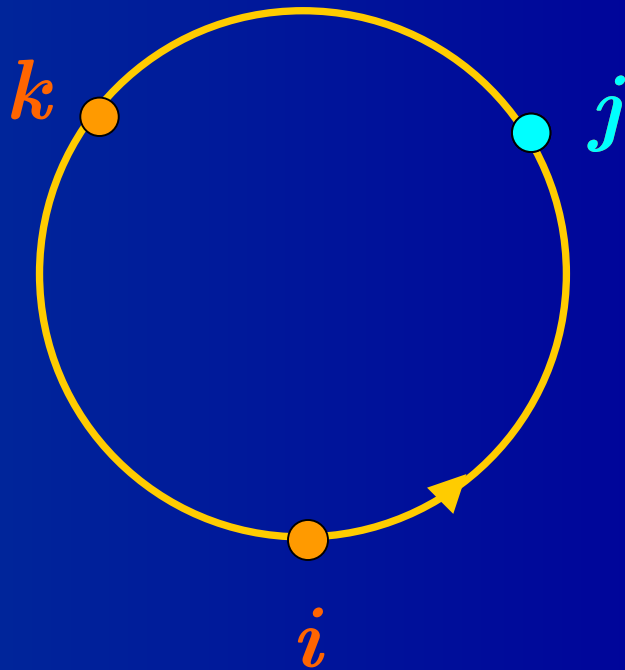
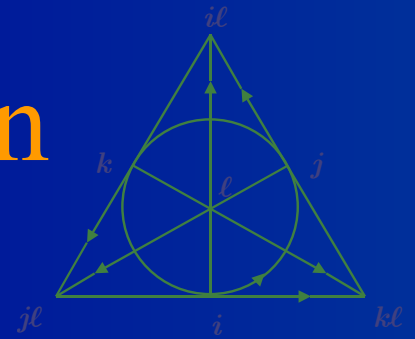


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Quaternionic Multiplication

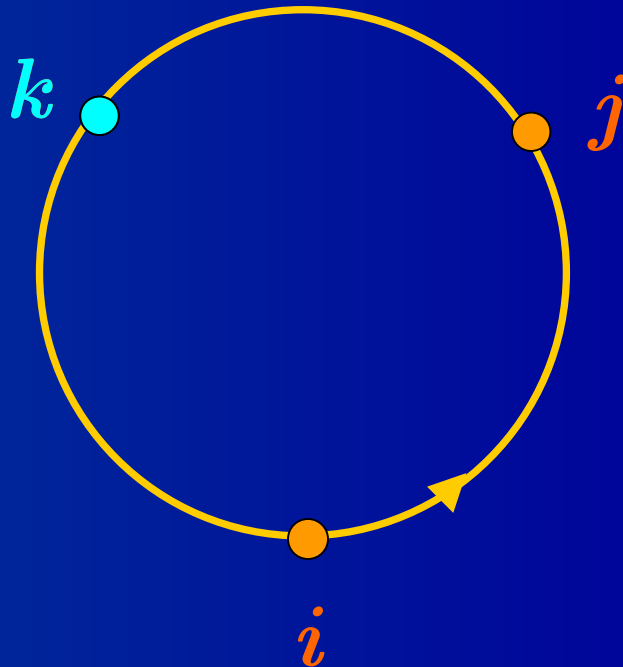
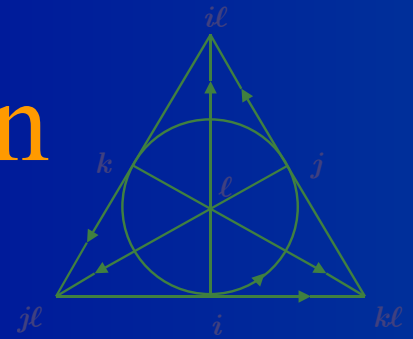


$$i^2 = j^2 = k^2 = -1$$

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Quaternionic Multiplication

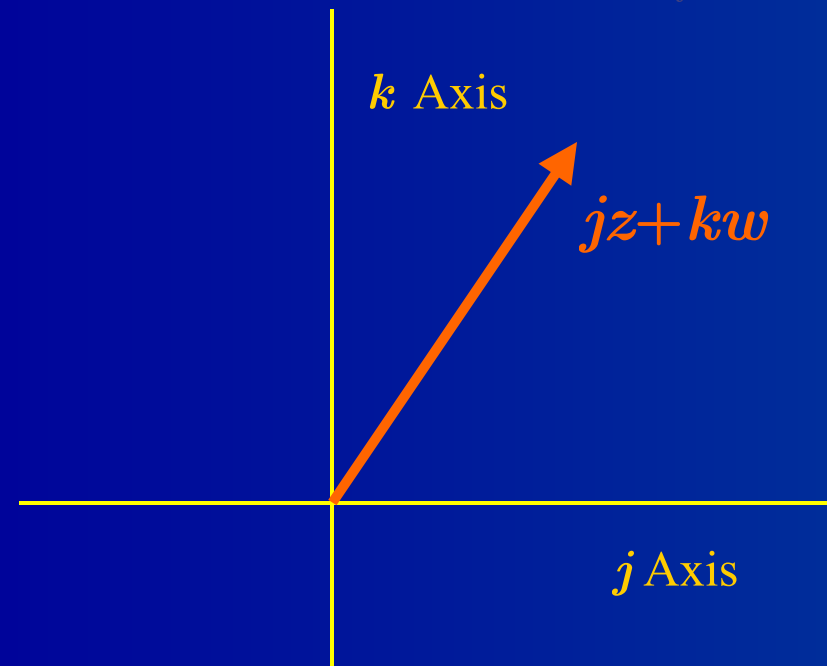
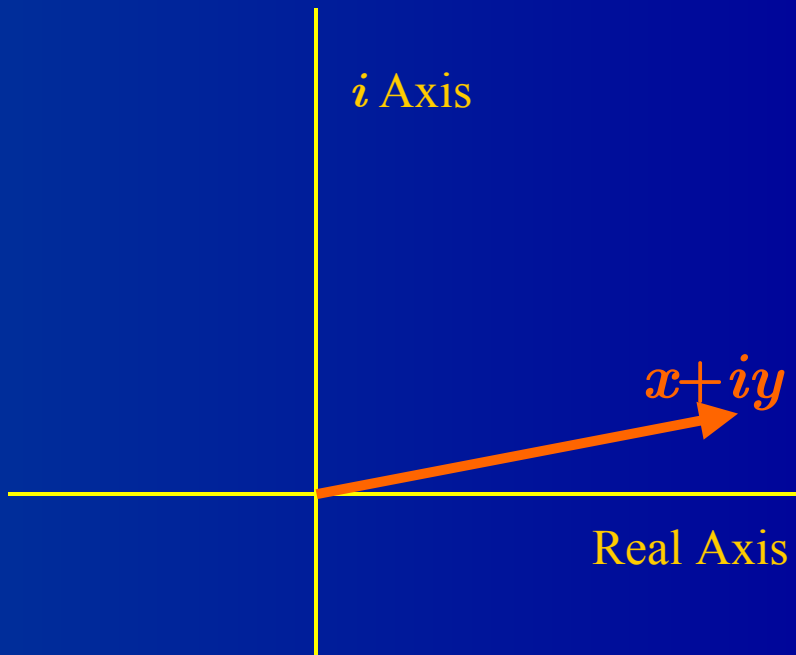
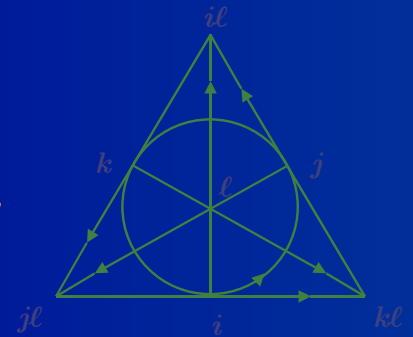


$$i^2 = j^2 = k^2 = -1$$

$$i j = k$$

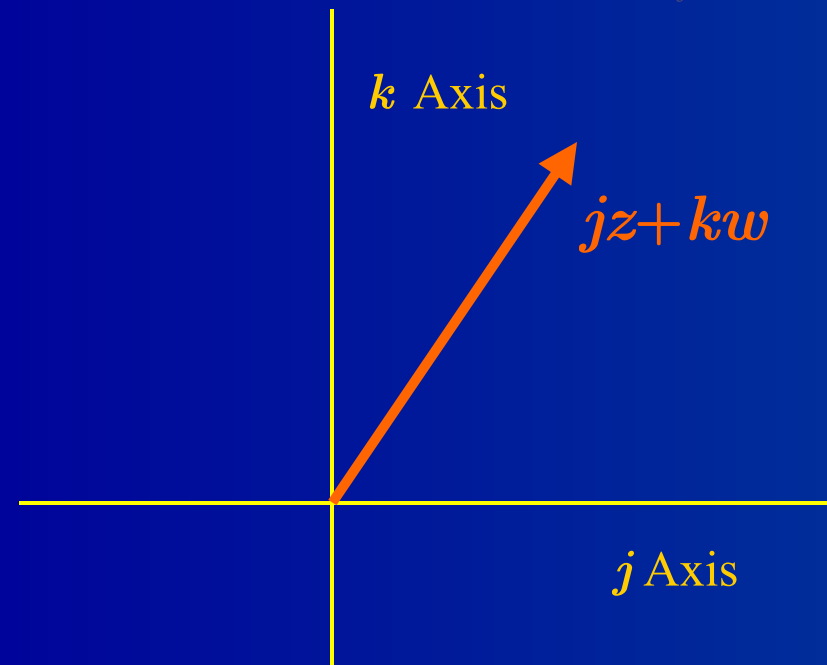
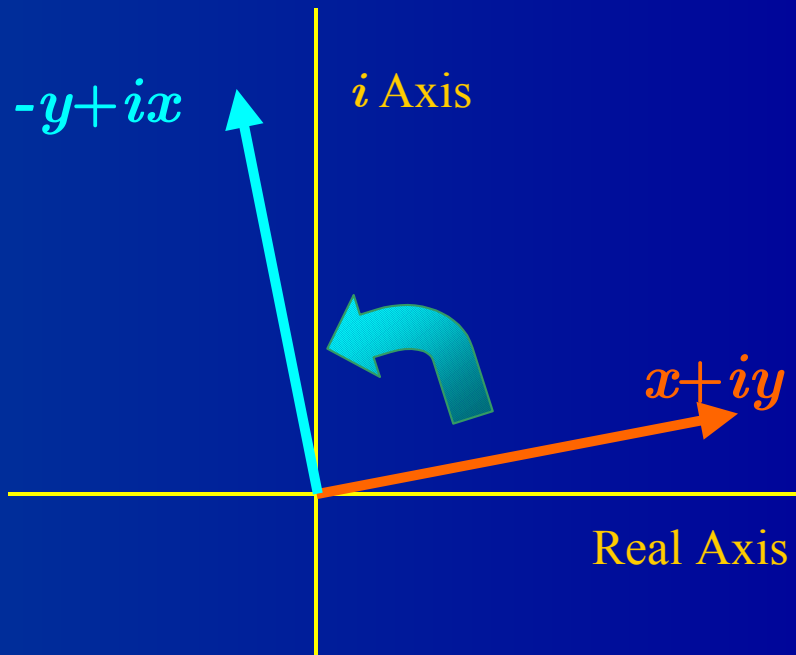
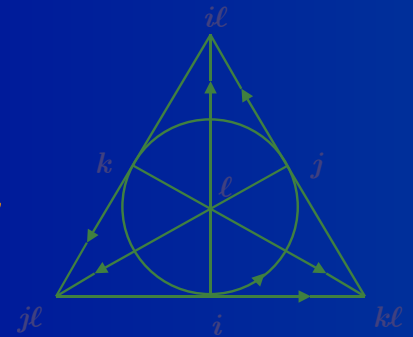
$$j i = -k$$

Left Multiplication by i



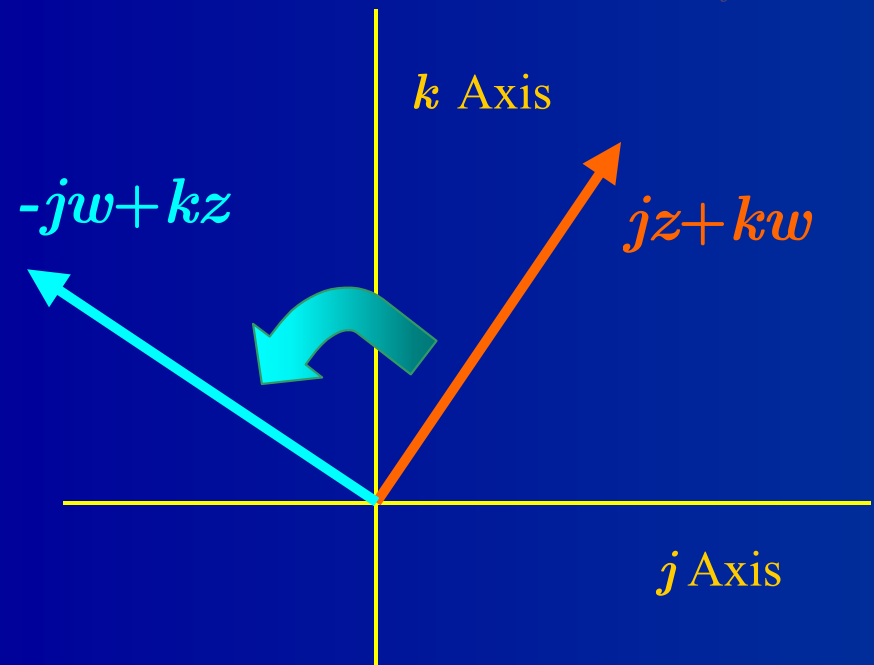
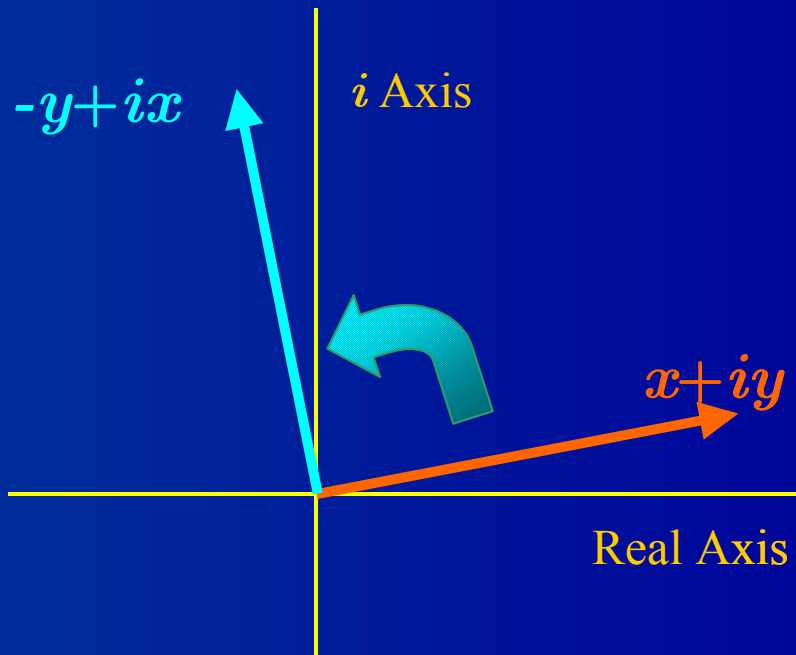
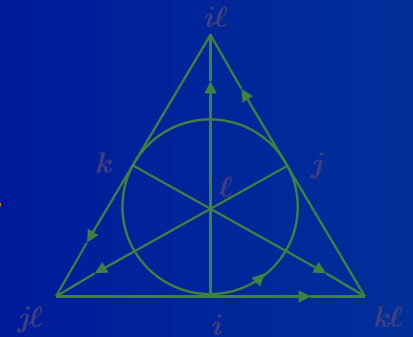
$$i(x + iy + jz + kw) = -y + ix - jw + kz$$

Left Multiplication by i



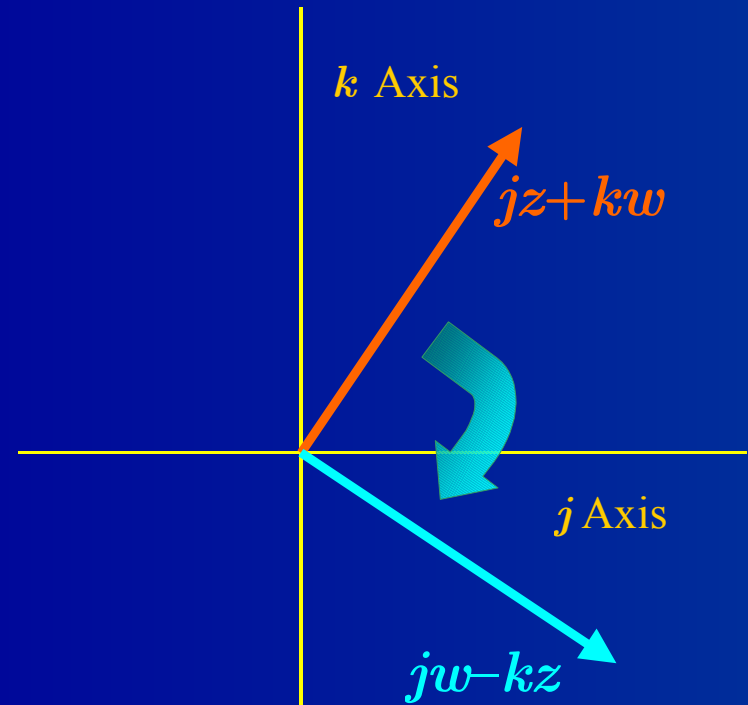
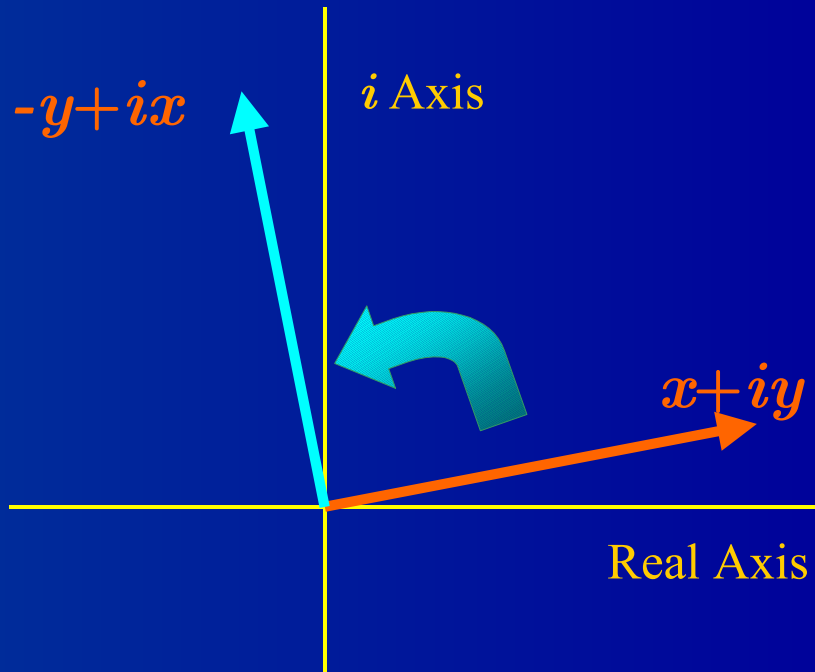
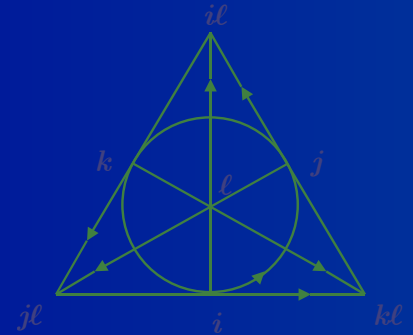
$$i(x + iy + jz + kw) = -y + ix - jw + kz$$

Left Multiplication by i



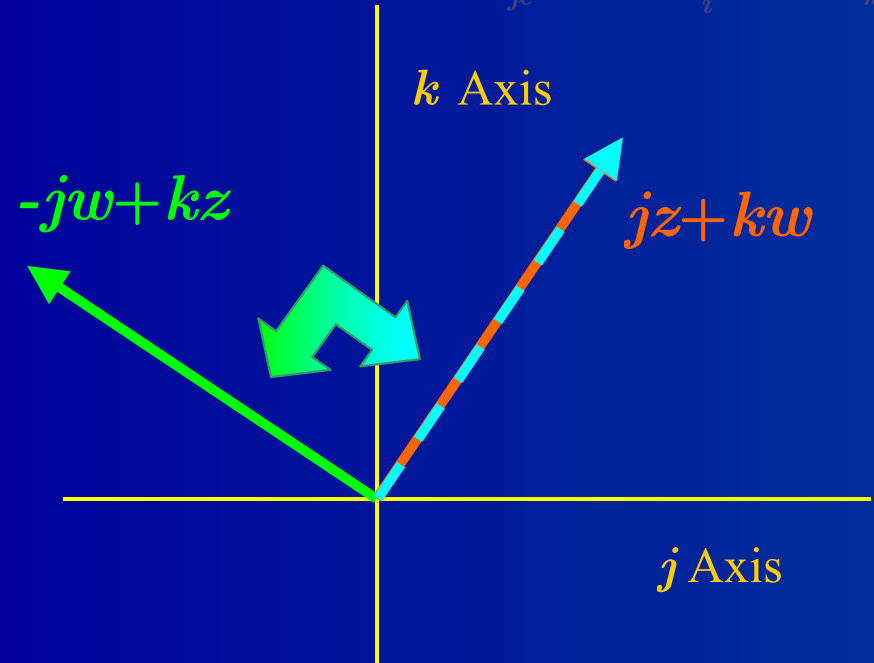
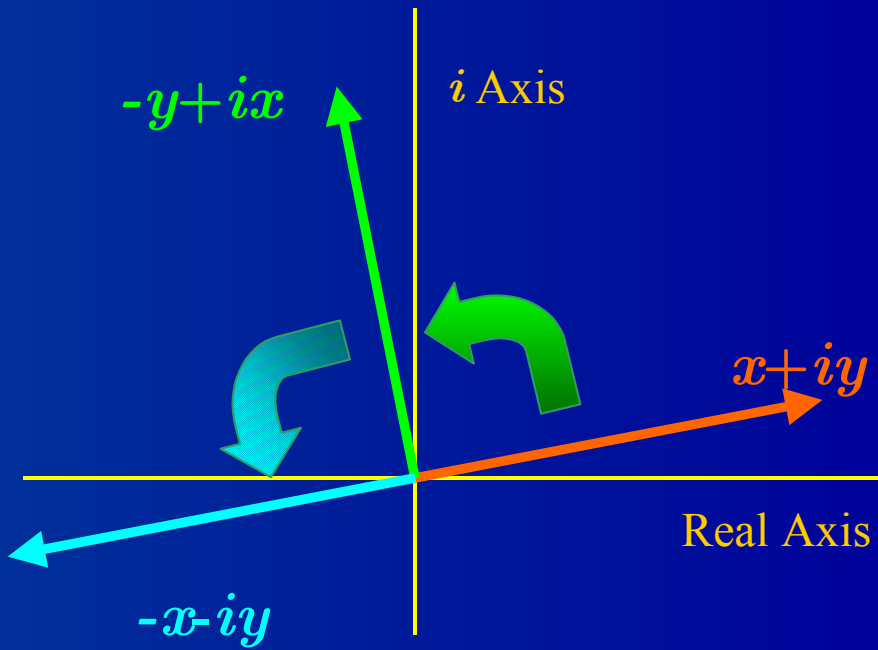
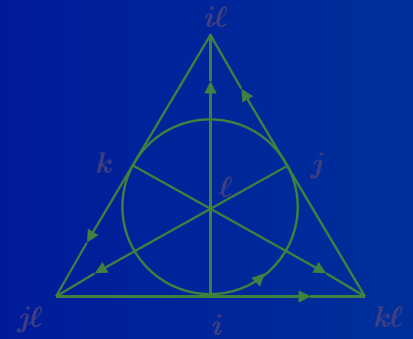
$$i(x + iy + jz + kw) = -y + ix - jw + kz$$

Right Multiplication by i



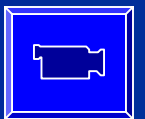
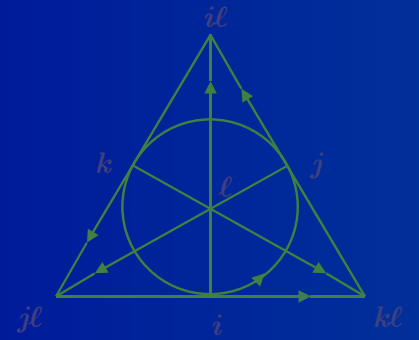
$$(x + iy + jz + kw)i = -y + ix + jw - kz$$

Flips by i

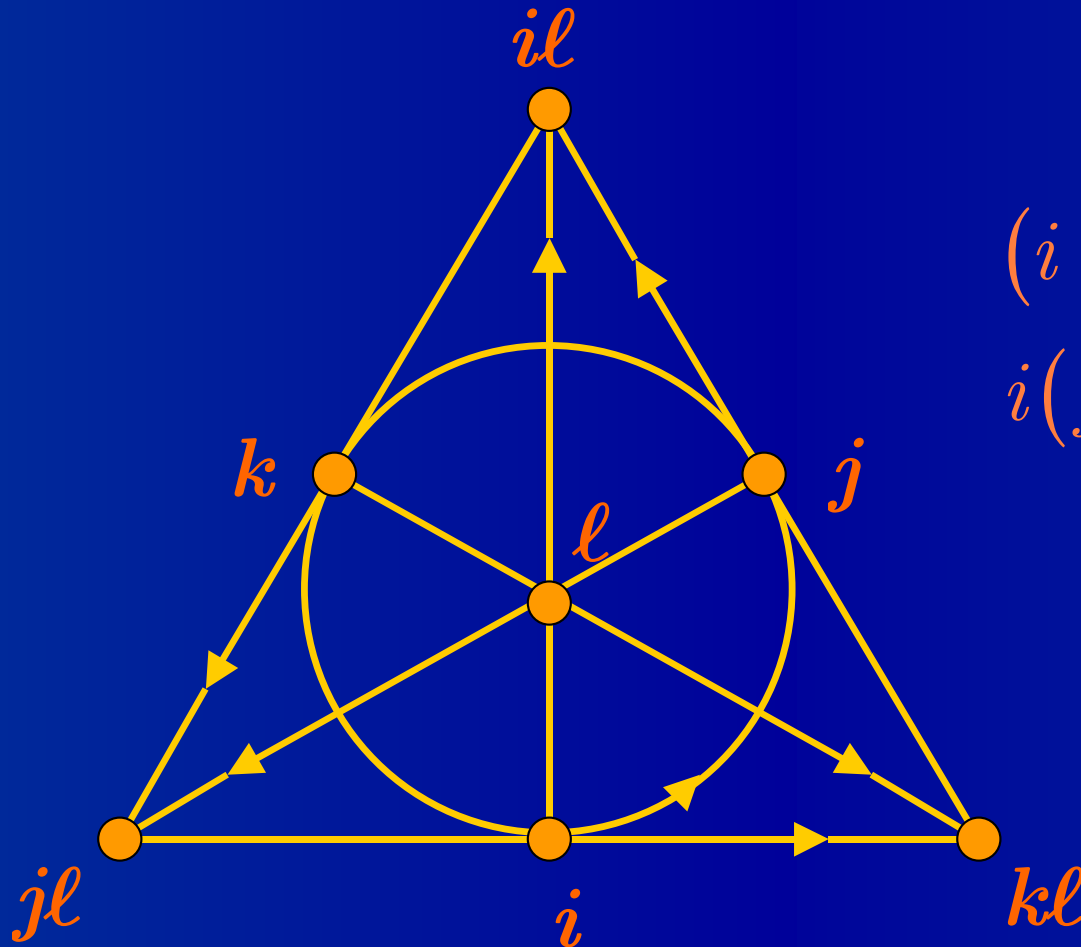
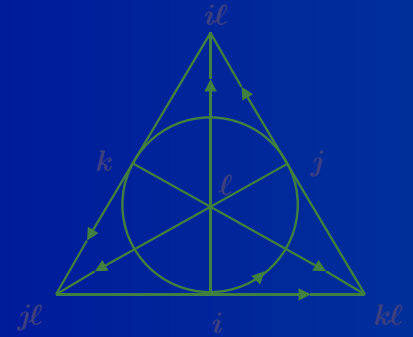


$$i(x + iy + jz + kw)i = -x - iy + jz + kw$$

Flips



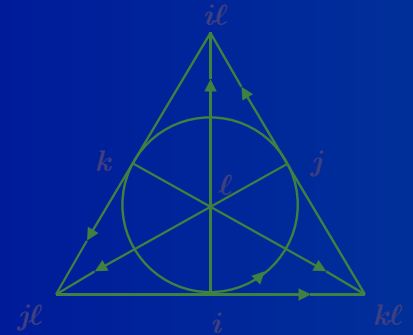
Octonionic Multiplication



$$(i \ j)\ell = k \ \ell = k\ell$$

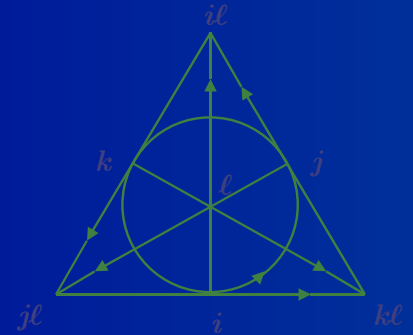
$$i(j \ \ell) = i \ j\ell = -k\ell$$

Fundamental Particles



Leptons						
Particle	e	ν_e	μ	ν_μ	τ	ν_τ
Charge	-1	0	-1	0	-1	0
Mass	0.511	0?	106	0?	1777	0?
Quarks						
Particle	u	d	c	s	t	b
Charge	2/3	-1/3	2/3	-1/3	2/3	-1/3
Mass	4.2	7.5	1000	150	180000000	4200

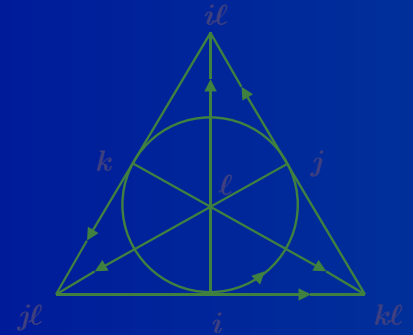
Composite Particles



Particle Charges	Anti-Particle Charges
$2/3$	$-2/3$
$-1/3$	$1/3$

- Mesons—
1 particle &
1 antiparticle
- Baryons—
3 particles

Forces and Mediators



- Four Forces each with “mediator” particle

- Electromagnetic

- photon—U(1)

- Weak

- W^{\pm} , Z—SU(2)

- Strong

- gluons—SU(3)

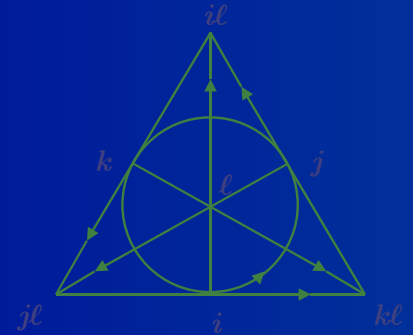
- Gravitational

- gravitons?



LIGO: The Hanford Interferometer

Equations for Leptons



- The Dirac Equation

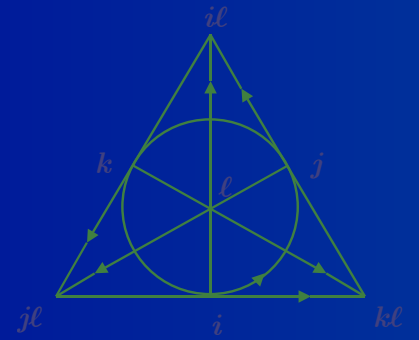
$$\sum_{\mu=0}^3 \left(p_{\mu} \gamma^{\mu} - m \right) \psi = 0$$

- The octonionic Dirac Equation

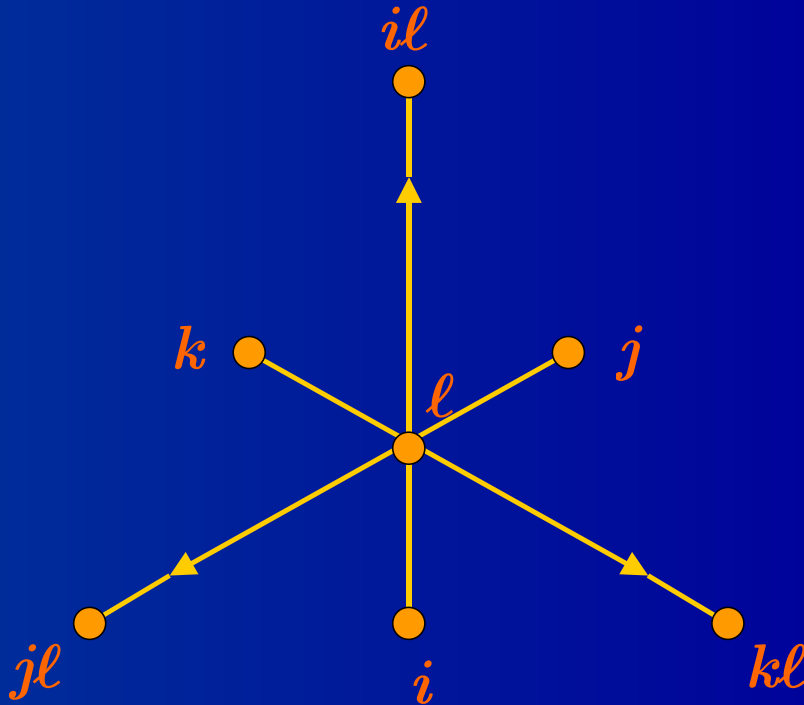
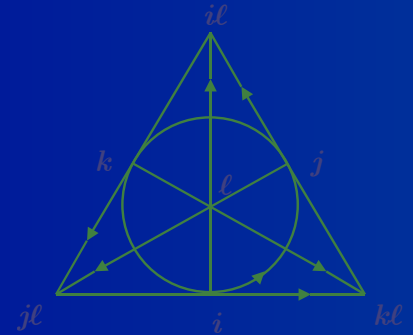
$$\tilde{\Pi} \psi = 0$$

- Involves 2x2 matrices
 - 10 spacetime dimensions
 - Naturally invariant under rotations and boosts

Spinors

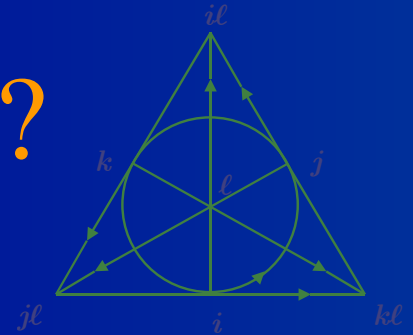


Leptons



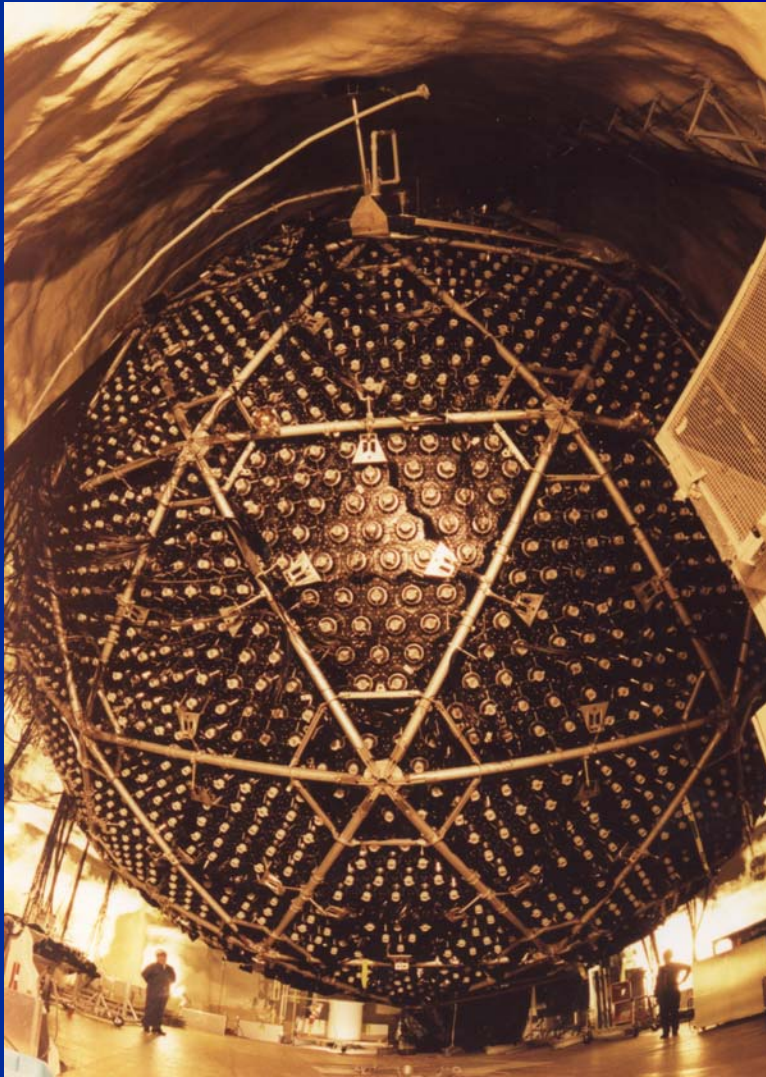
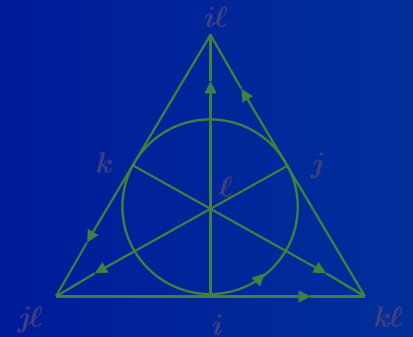
- Rotational symmetries are correct.
- Solutions are quaternionic.
- 3 generations!
- Massless neutrinos.

Where do we go from here?



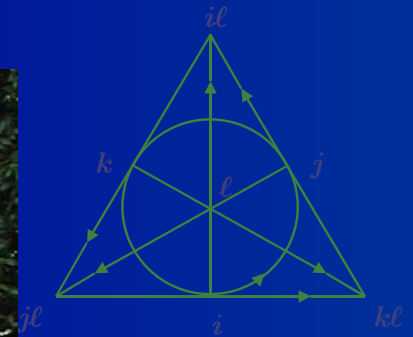
- Quarks
 - 3x3 octonionic matrices
- Interactions
 - E_6 contains $SU(3) \times SU(2) \times U(1)$
- Charges
- Masses

Neutrino Oscillations



Generation mixing
from curvature
instead of neutrino
mass?

Photo courtesy of Ernest Orlando Lawrence Berkeley National Laboratory



“The most incomprehensible thing about the universe is that it is comprehensible.”

– Einstein