THE BORDER BETWEEN RELATIVITY AND QUANTUM THEORY

Oregon State University OSU

Tevian Dray

I: Attempts at Unification
II: Spinors
III: The Future

Abstract

Many efforts have been made to fulfill Einstein's dream of unifying general relativity and quantum theory, including the study of quantum field theory in curved space, supergravity, string theory, twistors, and loop quantum gravity. While all of these approaches have had notable successes, unification has not yet been achieved. After a brief tour of the progress which has been made, this talk will focus on the fundamental role played by spinors in several of these approaches, suggesting that spinors are the key to combining classical relativity with quantum physics.

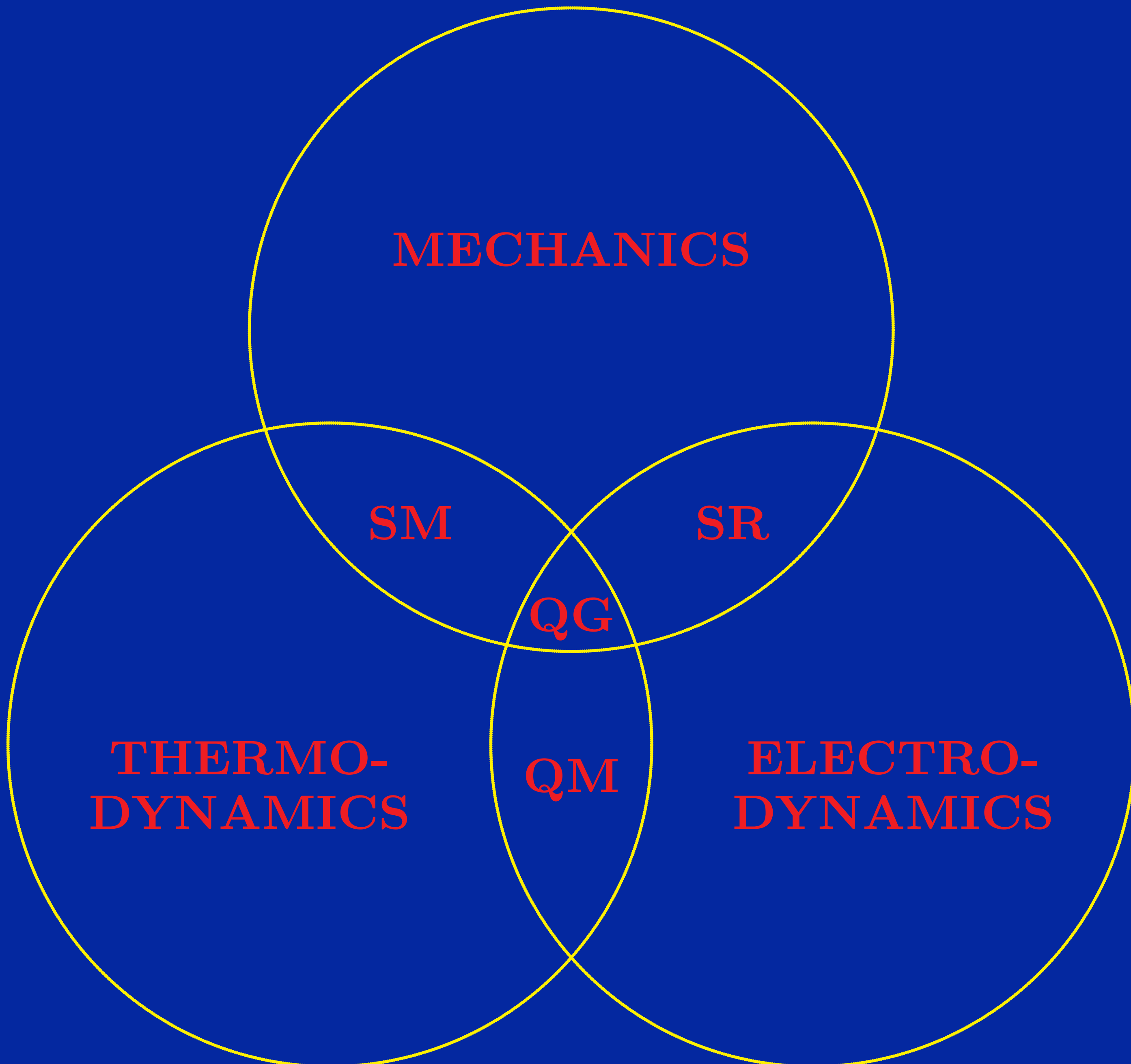
Apologies

- not much Spanish
- not really PowerPoint
- not many equations
- not historian

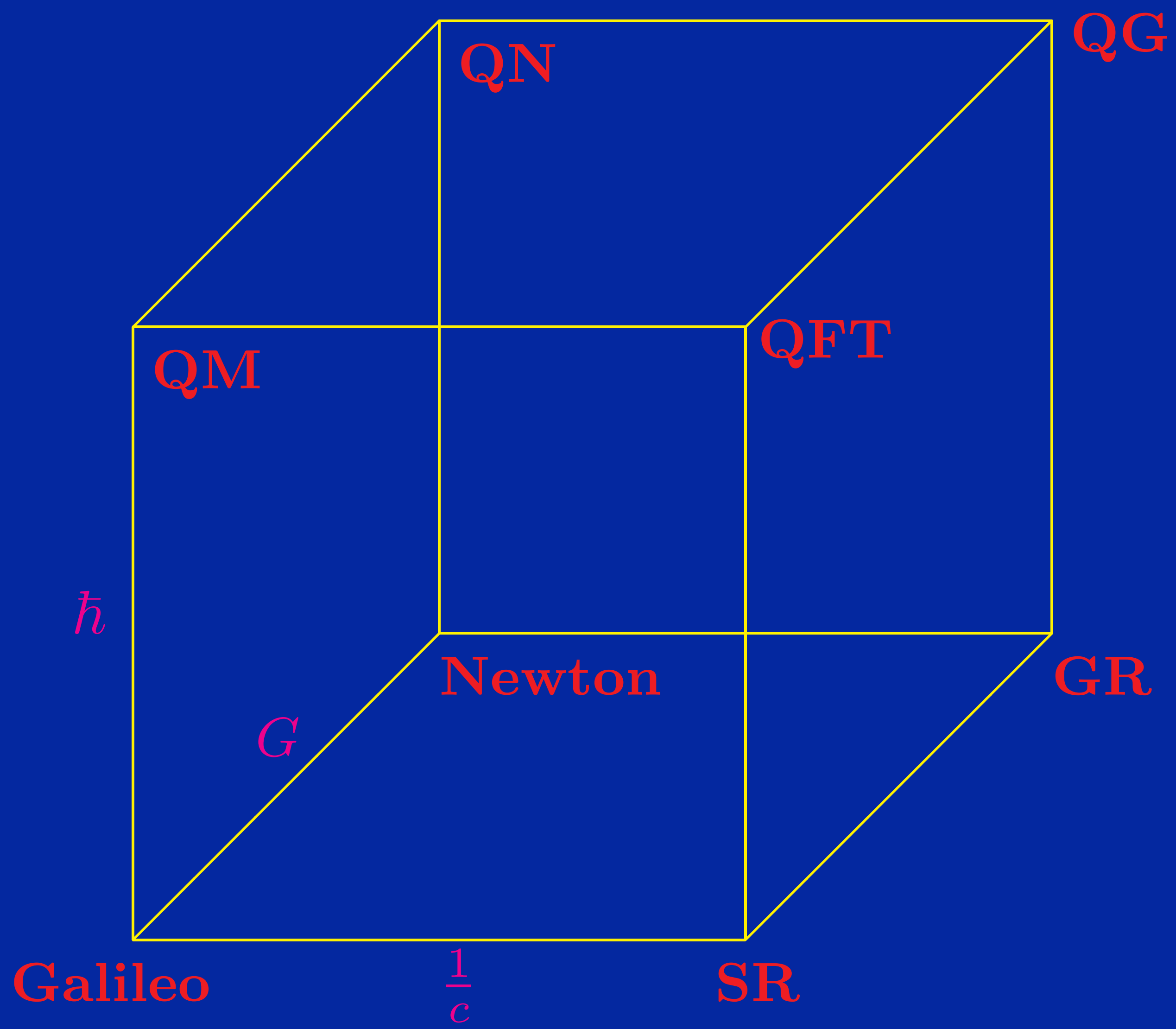
Acknowledgments

- Jürgen Renn
- Peter Bergmann
- Bryce DeWitt

Borderline Problems



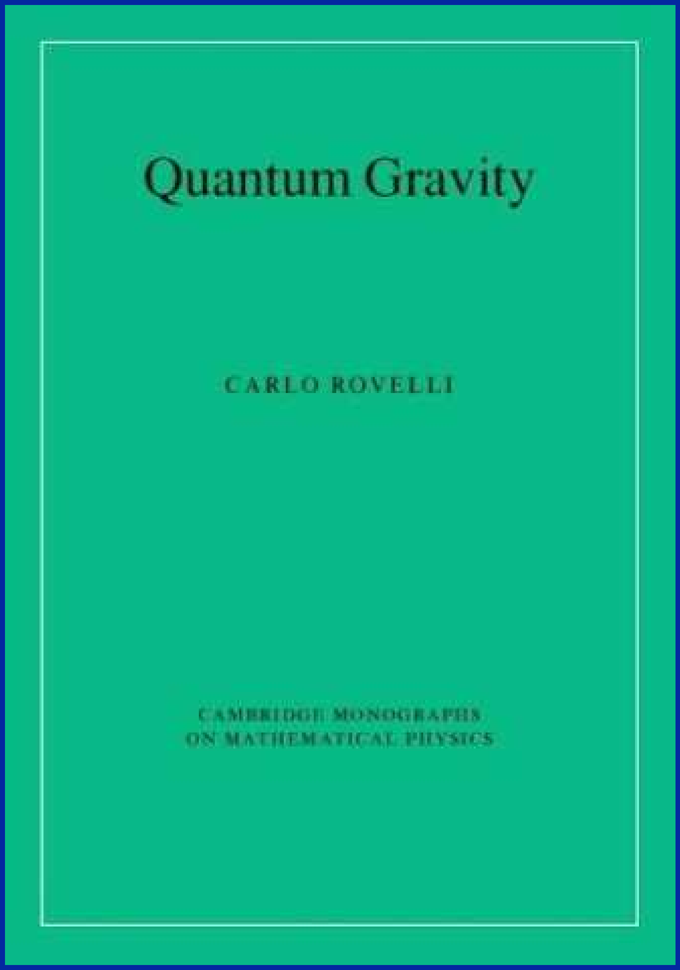
Bronstein Cube



All Roads Lead to Quantum Gravity

- QFT in curved space
- Canonical Quantum Gravity
- Loop Quantum Gravity
- Supergravity
- String Theory
- Twistors
- Other

Further Acknowledgment



Quantum Field Theory in Curved Space

Tevian Dray, Jürgen Renn, and Donald C. Salisbury, *Particle Creation with Finite Energy Density*, Lett. Math. Phys. **7**, 145–153 (1983).

- Fulling, Davies, Unruh, Ashtekar/Magnon:
  Particles are observer-dependent
- black hole thermodynamics
- not background independent
- no preferred vacuum state
- Wald: Essential ideas of QFT better identified in
  curved space than in Minkowski space!
- Answer: algebraic approach

Canonical Quantum Gravity

- Bergmann, Dirac: constraint algebra
- Arnowitt, Deser, Misner: new variables $(h_{ab},\, \pi^{ab})$
- Ashtekar: New Variables $(A_a{}^i,\, \widetilde{e}^{\,a}{}_i)$
  polynomial constraints
- Wheeler, DeWitt: formal field equation (WDW)

- time is frozen (Kuchař: time map)
- Salisbury: time-dependent gauge (Bergmann-Komar)

Loop Quantum Gravity

- Jacobson, Smolin: Loop-like solutions of WDW
- spatial discreteness natural
- background independent!
- no ultraviolet divergences
- possible control of singularities
- explains black hole entropy

- frozen time? (Rovelli: "partial observables")
- Lorentz invariance?
- semiclassical limit?
- scattering amplitudes?

Supergravity

- Kaluza-Klein theory for gravity: 11-dimensional
- covariant: QFT of metric fluctuations
- particle physics approach
- DeWitt, Feynman: Feynman rules for GR
- 't Hooft, Veltman, Deser, van Niuewenhuizen:
  non-renormalizable
- supergravity: renormalizable extension?

String Theory

$$\Pi^\mu \Pi_\mu = 0 \qquad \Pi^\mu \partial_\mu \theta = 0$$

- covariant: natural successor to supergravity
- supersymmetry: 3, 4, 6, 10 dimensions
- explains black hole entropy
- "GR at low energies"

- "GR at low energies"
- no supersymmetric particles yet!
- not (yet) background independent
- too many Calabi-Yau spaces

Twistors

$$Z^\alpha = (\omega^A, \pi_{A'}) \qquad [Z^\alpha, \overline{Z}_\beta] = \hbar\, \delta^\alpha{}_\beta$$

- combinatorics of null angular momentum
- complex numbers crucial to quantum mechanics
- complex numbers describe null directions
- non-locality: local info encoded in global fields

Wilczek: *"… at present twistor ideas appear more as the desire for a physical theory than the embodiment of one."*

Other

- Finkelstein: chronons; Clifford statistics
- Sorkin: causal sets
- Connes: noncommutative geometry
- Hawking: sum over histories; wave function of universe
- Penrose: gravity-induced quantum state reduction

Status

- Are there any observable consequences to a quantum theory of gravity?

  Major: *Some constraints, primarily from particle production rates at astrophysical energy scales, which are sensitive to Planck scale modifications in dispersion relations.*

- Burgess: no crisis: details of QG open, but effective theory of gravity known.

What's Missing?

- Gravity really is different.
- The essence of relativity is the lightcone structure.
- We need a revolutionary idea.

Vectors

$$\boldsymbol{x} = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \quad \longleftrightarrow \quad \boldsymbol{X} = \begin{pmatrix} t+z & x-iy \\ x+iy & t-z \end{pmatrix}$$

$$\boldsymbol{X}^\dagger = \overline{\boldsymbol{X}}^T = \boldsymbol{X}$$

$$-\det(\boldsymbol{X}) = -t^2 + x^2 + y^2 + z^2$$

$$\boldsymbol{X} = tI + x\sigma_x + y\sigma_y + z\sigma_z \ \text{(Pauli matrices)}$$

$$SO(3,1) \approx SL(2,\mathbb{C})$$

Clifford Algebras

Baylis: *Classical relativistic physics in Clifford's geometric algebra has a spinorial formulation that is closely related to the standard quantum formalism. The algebraic use of spinors and projectors, together with the bilinear relations of spinors to observed currents, gives quantum-mechanical form to many classical results, and the clear geometric content of the algebra makes it an illuminating probe of the quantum/classical interface.*

Division Algebras

Real Numbers:

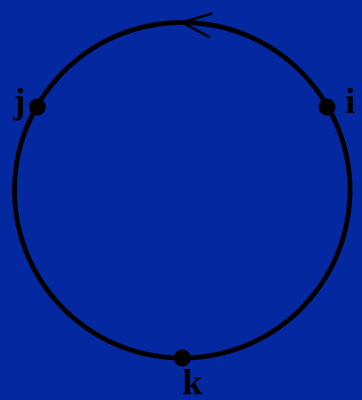
$$\mathbb{R}$$

Complex Numbers:

$$\mathbb{C} = \mathbb{R} + \mathbb{R}i$$

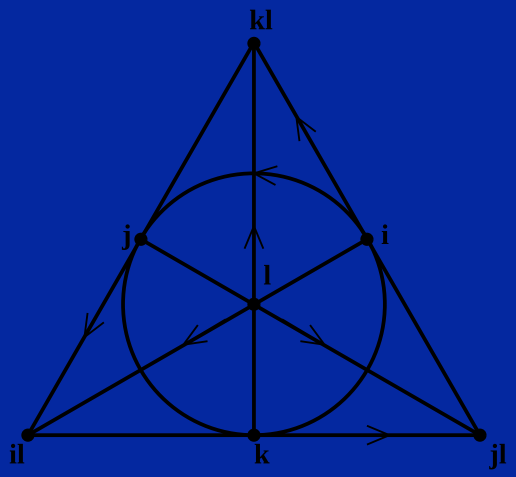
Quaternions:

$$\mathbb{H} = \mathbb{C} + \mathbb{C}j$$



Octonions:

$$\mathbb{O} = \mathbb{H} + \mathbb{H}\ell$$



More Vectors

$$SO(5,1) \approx SL(2,\mathbb{H})$$
$$SO(9,1) \approx SL(2,\mathbb{O})$$

$\mathbb{K} = \mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O} \longmapsto$

$$\boldsymbol{X} = \begin{pmatrix} p & \overline{a} \\ a & m \end{pmatrix} \qquad (p, m \in \mathbb{R}; a \in \mathbb{K})$$

$\dim \mathbb{K} + 2 = 3, 4, 6, 10$ spacetime dimensions

supersymmetry

Spinors

(Penrose, not Dirac)

$$v = \begin{pmatrix} \bar{b} \\ c \end{pmatrix}$$

$$vv^\dagger = \begin{pmatrix} |b|^2 & \bar{b}\bar{c} \\ cb & |c|^2 \end{pmatrix}$$

$$\det(vv^\dagger) = 0$$

$$(\text{spinor})^2 = \text{null vector}$$

Spinors

- Twistors:

  ω^A, $\pi_{A'}$ are complex spinors.

- String Theory:

  θ is a division algebra spinor;

  $\Pi = \theta\theta^\dagger - \theta^\dagger\theta.$

- Loop Quantum Gravity:

  ${A_a}^i$ is spin connection

Projections of higher-dimensional null vectors are null or timelike!

3 generations of leptons?

Jordan Algebras

$$\mathcal{X} = \begin{pmatrix} p & a & \overline{b} \\ \overline{a} & m & c \\ b & \overline{c} & n \end{pmatrix}$$

$$\mathcal{X} \circ \mathcal{Y} = \frac{1}{2} (\mathcal{X}\mathcal{Y} + \mathcal{Y}\mathcal{X})$$

Exceptional quantum mechanics:

$$(\mathcal{X} \circ \mathcal{Y}) \circ \mathcal{X}^2 = \mathcal{X} \circ (\mathcal{Y} \circ \mathcal{X}^2)$$

P. Jordan, J. von Neumann, and E. Wigner, *On an Algebraic Generalization of the Quantum Mechanical Formalism*, Ann. Math. **35**, 29–64 (1934).

“Superspinors”

$$\mathcal{X} = \begin{pmatrix} \mathbf{\Pi} & \theta \\ \theta^\dagger & \phi \end{pmatrix}$$

Cayley/Moufang plane:

$$\begin{aligned} \mathbb{OP}^2 &= \{\mathcal{X}^2 = \mathcal{X}, \operatorname{tr}\mathcal{X} = 1\} \\ &= \{\mathcal{X} = \psi\psi^\dagger, \psi^\dagger\psi = 1, \psi \in \mathbb{H}^3\} \end{aligned}$$

$$\det(\mathcal{M}\mathcal{X}\mathcal{M}^\dagger) = \det\mathcal{X} \Longrightarrow \mathcal{M} \in E_6$$

$$SO(3,1) \times U(1) \times SU(2) \times SU(3) \subset E_6$$

Where’s gravity?? *“Jordan” Spin Foam?*

All Roads Lead to Quantum Gravity

The current theories are on the right track.

(But do they know it?)

THE END

Octonion References

Corinne A. Manogue and Tevian Dray, *Dimensional Reduction*, Mod. Phys. Lett. **A14**, 93–97 (1999).

Corinne A. Manogue and Tevian Dray, *Octonionic Möbius Transformations*, Mod. Phys. Lett. **A14**, 1243–1255 (1999).

Tevian Dray and Corinne A. Manogue, *The Exceptional Jordan Eigenvalue Problem*, Internat. J. Theoret. Phys. **38**, 2901–2916 (1999).

Tevian Dray and Corinne A. Manogue, *Quaternionic Spin*, in **Clifford Algebras and their Applications in Mathematical Physics**, eds. Rafał Abłamowicz and Bertfried Fauser, Birkhäuser, Boston, 2000, pp. 29–46.

http://www.math.oregonstate.edu/~tevian