Integrals in Mathematics and Physics

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Mathematics vs. Physics

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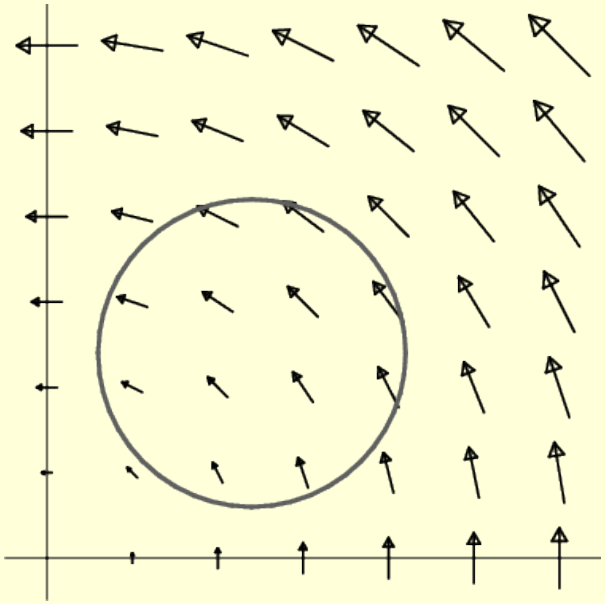
$$\vec{\mathrm{F}} = \langle P, Q, R \rangle \qquad \text{vs.} \qquad \vec{\mathrm{F}} = F_x\,\hat{\mathrm{x}} + F_y\,\hat{\mathrm{y}} + F_z\,\hat{z}$$

Vector Line Integrals: $\int \vec{F} \cdot d\vec{r}$



Research Question:

- What does an analysis of textbook treatments of vector line integrals reveal about the learning objectives and (abbreviated) learning trajectories of the associated courses?

Vector Line Integrals in Mathematics and Physics, IJRUME **9**, 92–117 (2023)

Representational Transformation Diagram (RTD)

A flowchart to represent and analyze rich concept images.
Bajracharya, Emigh, and Manogue, Phys. Rev. Phys. Educ. Res. **15**, 020124 (2019).

- *Translation* (one arrow in and out)
- *Consolidation* (≥ 1 arrows in)
- *Dissociation* (≥ 1 arrows out)

Length and complexity of RTD is proxy for cognitive load.

Look for:

- *Iconic expression or equation*;
- How the iconic expression is *unpacked*;
- What is the *starting point for calculation*.

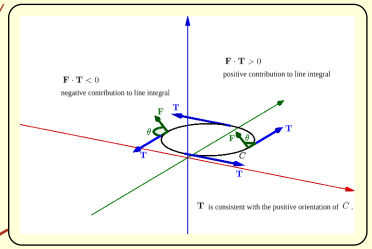
Iconic expression: the symbolic representation of a fundamental concept in its simplest, most compact form.

- *Geometric* (independent of origin, coordinates, parameterization);
- *Easy to remember*;
- Contains instructions for *unpacking* in different contexts.

Mathematics

Riemann Sum

$$\int_C f\, ds$$



$$\int_C \mathsf{F} \cdot \mathsf{T}\, ds$$

$$\mathsf{T} = \frac{\mathsf{r}'(t)}{|\mathsf{r}'(t)|}$$

$$\mathsf{r}(t) = \langle x(t), y(t), z(t) \rangle$$

$$ds = |\mathsf{r}'(t)|\, dt$$

$$\int_C \mathsf{F} \cdot \mathsf{r}'(t)\, dt$$

$$\mathsf{F} = \langle f, g, h \rangle$$

$$dx = x'(t)\, dt, \ldots$$

$$d\mathsf{r} = \langle dx, dy, dz \rangle$$

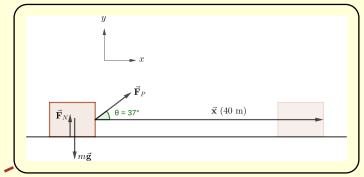
$$\int_a^b \left(f(t)x'(t) + \ldots\right)\, dt$$

$$\int_C f\, dx + g\, dy + h\, dz$$

$$\int_C \mathsf{F} \cdot d\mathsf{r}$$

Physics

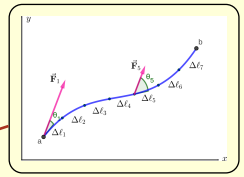
$W = Fd$



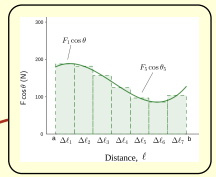
$W = F_{\|} d = Fd \cos\theta$

$\vec{A} \cdot \vec{B} = AB \cos\theta$

$W = \vec{F} \cdot \vec{d} = Fd \cos\theta$



$W \approx \sum F_i \cos\theta_i \Delta\ell_i$



$W = \int_a^b F \cos\theta \, d\ell$

$d\ell = |d\vec{\ell}|$ (in words)

$W = \int_a^b \vec{F} \cdot d\vec{\ell}$

$\vec{F} = F_x \hat{\imath} + \ldots$

$\vec{A} \cdot \vec{B} = A_x B_x + \ldots$

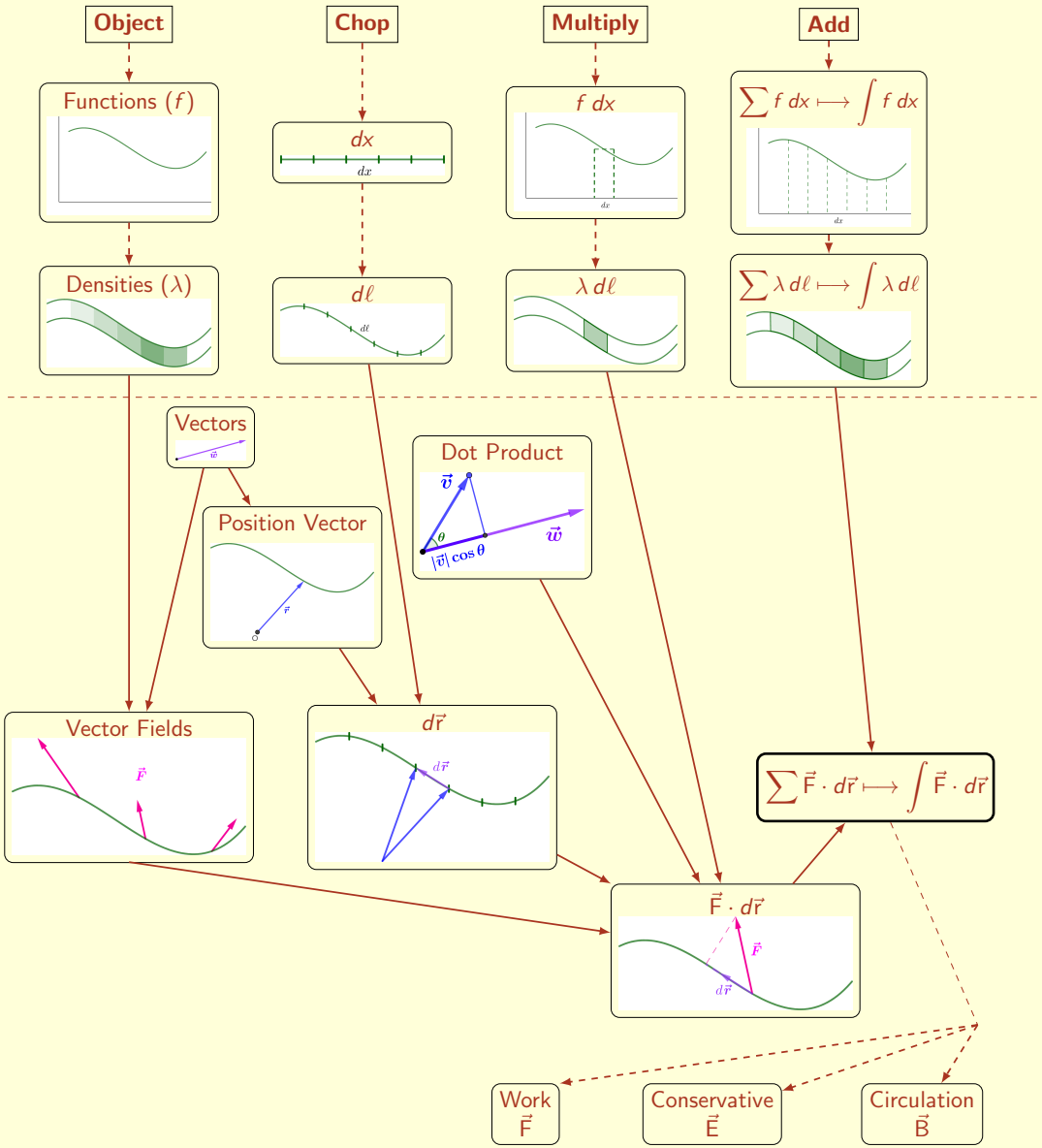
$d\vec{\ell} = dx \hat{\imath} + \ldots$

$W = \int_{x_a}^{x_b} F_x \, dx + \ldots$

Jones (2020); Pina & Loverude (2019):

Way of thinking	Integrals are interpreted as ...
Space underneath a graph	... the amount of space underneath the graph
Antiderivative	... an instruction to compute an antiderivative
Adding up pieces	... the summation of infinitesimal quantities
Accumulation from rate	... the accumulation from a rate function
Averaging	... an averaging across the domain
Procedural	... an operator to further a derivation

Suggested learning trajectory



Chop, Multiply, Add

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https://bridge.math.oregonstate.edu/papers/IJRUMEintegrals.pdf

This work forms an integral part of the Paradigms in Physics project, incorporating also the originally separate Vector Calculus Bridge project. These projects have been supported by NSF grants 0088901, 0231032, 0618877, 1023120, 1256606, 1323800, 1836603, and 1836604.

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