$$\mathbb{R}^3(\bar{*}, \bar{d}) \quad \text{vs} \quad \mathbb{M}^4(*, d)$$

$$\bar{\omega} = dx \wedge dy \wedge dz \qquad \omega = \bar{\omega} \wedge dt$$

$$\Rightarrow df = \bar{d} f + \dot{f}\, dt$$

$$d\alpha = \bar{d}\alpha + dt \wedge \dot{\alpha}$$

$$* E = (\bar{*} E) \wedge dt$$

$$* f\, dt = \bar{*} f$$

(III) differential forms in $\mathbb{M}^4$

$$\boxed{F = \bar{E} \wedge dt + \bar{*}\bar{B}}$$

$$\Rightarrow \; * F = \bar{B} \wedge dt - \bar{*}\bar{E}$$

$$\therefore \; dF = d\bar{E} \wedge dt + d\bar{*}\bar{B}$$

$$= (\bar{d}\bar{E} + \cancel{dt \wedge \dot{\bar{E}}}) \wedge dt + (\cancel{\bar{d}\bar{*}\bar{B}} + dt \wedge \bar{*}\dot{\bar{B}})$$

$$= -\bar{*}\dot{\bar{B}} \wedge dt + dt \wedge \bar{*}\dot{\bar{B}} = 0$$

$$d * F = d\bar{B} \wedge dt - d\bar{*}\bar{E}$$

$$= \bar{d}\bar{B} \wedge dt - (\bar{d}\bar{*}\bar{E} + dt \wedge \bar{*}\dot{\bar{E}})$$

$$= (\bar{d}\bar{B} - \bar{*}\dot{\bar{E}}) \wedge dt - \bar{d}\bar{*}\bar{E}$$

$$= 4\pi(\bar{*}\bar{J} \wedge dt - \bar{*}\rho)$$

$$= 4\pi(*\bar{J} - *\rho\, dt) =: 4\pi * J$$

Ansatz: $\boxed{A = \bar{A} - \Phi\, dt}$ $\quad (J = \bar{J} - \rho\, dt)$

$$\Rightarrow dA = d\bar{A} - d\Phi \wedge dt$$

$$= \bar{d}\bar{A} + dt \wedge \dot{\bar{A}} - \bar{d}\Phi \wedge dt$$

$$= -(\bar{d}\Phi + \dot{\bar{A}}) \wedge dt + \bar{d}\bar{A}$$

$$= -\bar{E} \wedge dt + \bar{*}\bar{B} = F$$

$$\therefore \; \boxed{\begin{aligned} F &= dA \\ d * F &= 4\pi * J \end{aligned}}$$