Vector Spaces

$\vec{u}, \vec{v}, \vec{w} \in V$
$a, b \in \mathbb{R}$

2 operators : $+$, scalar mult

closed under both

commutative

$$\vec{0} + \vec{v} = \vec{v}$$

$$\exists! (-\vec{v}) : \vec{v} + (-\vec{v}) = \vec{0}$$

Compatibility : $(ba)(\vec{v}) = b(a\vec{v})$

Linearity : $a(\vec{v} + \vec{w}) = a\vec{v} + a\vec{w}$
$(a+b)\vec{v} = a\vec{v} + b\vec{v}$

Identity : $0\vec{v} = \vec{0}$
$1\vec{v} = \vec{v}$

Tell me something you know about an inner product.

Notation: $\vec{u}\cdot\vec{v}$ $\quad u^T v$, $u^\dagger v$

$\langle u|v\rangle$ $\quad \langle u, v\rangle$

$h(\vec{u}, \vec{v})$

Properties: $\quad h : V \times V \to \mathbb{R}$

- bilinear (linear in each dot)
- symmetric $\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}$
- nondegenerate: $\vec{v}\cdot\vec{w} = 0 \ \forall \vec{w} \implies \vec{v} = \vec{0}$