MTH 679 **HW #6** Spring 2016

Recall from the previous assignment that the condition

$$2\,\frac{v \cdot w}{v \cdot v} \in \mathbb{Z} \qquad (*)$$

for all vectors in some subset of $\mathbb{R}^n$ tightly constrains both the angles between those vectors and their relative lengths. *All vectors throughout this assignment are assumed to lie in a subset $S \subset \mathbb{R}^n$ that satisfies the above constraint.*

1. (a) Choose two vectors $v, w \in \mathbb{R}^2$ satisfying (*) and such that the angle between them is $\frac{3\pi}{4}$.
   (b) Embedding $\mathbb{R}^2$ in $\mathbb{R}^3$, it is straightforward to choose $u \perp \mathbb{R}^2$ and to show that $S = \{u, v, w\} \subset \mathbb{R}^3$ satisfies (*). Find some *other*, linearly independent u, *not* perpendicular to both v and w, such that (*) still holds.
   (c) Is there more than one way to do this?
   (d) What are the angles between each pair of vectors in S? What are the ratios of their magnitudes?
   (e) Do your solution(s) satisfy the additional condition that

   $$v \cdot w \leq 0 \qquad (**)$$

   for all elements $v, w \in S$?

2. ***BONUS:*** Can you extend your solution to $\mathbb{R}^4$? What are the resulting angles and ratios? Is there more than one way to do this (starting from the same set in $\mathbb{R}^3$)?
   Your four vectors should be linearly independent, and none should be orthogonal to all three of the others.

3. ***CHALLENGE:*** Now suppose that the angle between $v, w \in \mathbb{R}^2$ is $\frac{5\pi}{6}$, and repeat Question 1a. Then attempt to repeat Question 1b.
   Do not spend too much time on this problem without consulting me!