

1. Suppose  $V \subset \mathbb{R}^n$  is a collection of vectors such that

$$2 \frac{v \cdot w}{v \cdot v} \in \mathbb{Z}$$

for all elements  $v, w \in V$ . ( $\mathbb{Z}$  denotes the integers;  $\cdot$  denotes the usual dot product.)

- (a) What are the possible angles between  $v$  and  $w$ ?
- (b) What are the possible ratios of  $|v|$  to  $|w|$ ?  
*You may assume  $|v| \leq |w|$ .*