HW #4

1. (a) Find an orthonormal basis of the Lie algebra $\mathfrak{sl}(3,\mathbb{R})$.
   Orthonormal means orthogonal with respect to the Killing form $B(X,Y) = \mathrm{tr}\,(XY)$*, and with each element having squared magnitude* ± 1.

   (b) What is the dimension of this Lie algebra? How many boosts are there?

   (c) Find two orthogonal elements of $\mathfrak{sl}(3,\mathbb{R})$ that commute with each other.

   (d) Find all simultaneous eigenvectors of these two elements *in* $\mathfrak{sl}(3,\mathbb{R})$.
   Don't forget which vector space you are in; the eigenvector equation has the form

   $$[Q, X] = \lambda X$$

   (e) Find a basis of $\mathfrak{sl}(3,\mathbb{R})$ consisting entirely of simultaneous eigenvectors of your commuting operators.

   (f) Compute all commutators of these basis elements.
   Can you rescale your basis so that the commutator coefficients are all integers?

   (g) Make a table of the pairs of eigenvalues associated with each basis element. Plot these points in $\mathbb{R}^2$. *(Do not use your rescaled basis here.)*

   (h) What are the angles between these points (measured at the origin)?

2. ***OPTIONAL:*** What Lie algebra is $\mathfrak{sl}(3,\mathbb{R})$ a real form of? Equivalently, what compact Lie algebra do you get if you replace all boosts by rotations?
   Yes, you can accomplish this transition by multiplying by i.