HW #3

1. Recall that $\mathbb{C}' = \mathbb{R} \oplus \mathbb{R}L$ where $L^2 = +1$. We can define unitary groups over $\mathbb{C}'$ by analogy with the usual construction over $\mathbb{C}$, yielding for instance

$$\mathrm{SU}(2, \mathbb{C}') = \{M \in \mathbb{C}'^{2\times 2} : M^\dagger M = 1, |M| = 1\}. \tag{1}$$

   (a) Find a basis for the Lie algebra $\mathfrak{su}(2, \mathbb{C}')$.

   (b) How many boosts are in your basis?

   (c) How many of the elements of your basis are Hermitian? anti-Hermitian?

   (d) Compute the commutators of your basis elements. Do you recognize the result?

   (e) What Lie algebra do you think $\mathfrak{su}(2, \mathbb{C}')$ is isomorphic to?

   (f) Verify that the corresponding Lie *group* is (locally) isomorphic to $\mathrm{SU}(2, \mathbb{C}')$.

2. ***CHALLENGE:*** Repeat the previous problem for $\mathrm{SU}(2, \mathbb{C}' \otimes \mathbb{C})$.

   ($\mathbb{C}' \otimes \mathbb{C}$ is the 4-dimensional algebra over $\mathbb{R}$ contining both $i^2 = -1$ and $L^2 = 1$, together with the assumption that i and L commute.)

   Definitely optional. Partial solutions and/or guesses are welcome.