# HW #1



1. Suppose $G$ is an $n \times n$ matrix. Define a "dot product" on $\mathbb{R}^n$ by

$$\boldsymbol{v} \cdot \boldsymbol{w} = \boldsymbol{v}^T G \boldsymbol{w}$$

   (a) What conditions on $G$ guarantee that this dot product is a symmetric, non-degenerate bilinear form on $\mathbb{R}^n$?

   (b) Are your conditions necessary as well as sufficient?

   (c) Suppose $T : \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation with matrix $M$. What condition on $M$ is equivalent to $T(\boldsymbol{v}) \cdot T(\boldsymbol{w}) = \boldsymbol{v} \cdot \boldsymbol{w}$ for all $\boldsymbol{v}, \boldsymbol{w} \in \mathbb{R}^n$?

2. Suppose $G$ is an $n \times n$ matrix. Define a "dot product" on $\mathbb{R}^n$ by

$$\boldsymbol{v} \cdot \boldsymbol{w} = \boldsymbol{v}^T G \boldsymbol{w}$$

   (a) What conditions on $G$ guarantee that this dot product is an anti-symmetric, non-degenerate bilinear form on $\mathbb{R}^n$?

   (b) Are your conditions necessary as well as sufficient?

   (c) Show that the conditions in (a) imply that $n$ is even.

3. (a) Compute $c'(0)$ if

$$c(\beta) = \begin{pmatrix} \cosh\beta & \sinh\beta \\ \sinh\beta & \cosh\beta \end{pmatrix}$$

   (b) Compute $\gamma'(0)$ if

$$\gamma(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   (c) Compute $\sigma'(0)$ if $\sigma(\alpha) = \gamma(\alpha)^2$ and compare your result with $\gamma'(0)$.