

1. Let $\{x, y\}$ and $\{r, \phi\}$ denote the usual rectangular and polar coordinate systems on \mathbb{R}^2 . Compute $dx \wedge dy$ in polar coordinates (using the corresponding coordinate basis for $\wedge^2 \mathbb{R}^2$).
2. Let $\{x, y\}$ be rectangular coordinates on \mathbb{R}^2 .
 - (a) Find coordinates $\{u, v\}$ on \mathbb{R}^2 such that $u = x$ but $\frac{\partial}{\partial u} \neq \frac{\partial}{\partial x}$.
 - (b) Explicitly determine the relationship between the bases $\{\frac{\partial}{\partial u}, \frac{\partial}{\partial v}\}$ and $\{\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\}$.
 - (c) Explicitly determine the relationship between the bases $\{dx, dy\}$ and $\{du, dv\}$.