MTH 674 **HW #3** Spring 2005

1. Consider the group $SO(3) = SO(3, \mathbb{R})$ of real orthogonal 3×3 matrices, that is, real 3×3 matrices M satisfying $MM^t = I = M^t M$.

(a) Write down the 1-parameter subgroups $R_i(\phi)$ of $SO(3)$ corresponding to (counterclockwise) rotations by ϕ about the axes $x^i = x, y, z$.

(b) Find the generators X_i of the action of each of these subgroups.
Your answers should be vector fields on $\mathbb{R}^3$.

(c) The *commutator* of vector fields is defined by

$$[X, Y](f) := X\Big(Y(f)\Big) - Y\Big(X(f)\Big)$$

Compute the commutators $[X_i, X_j]$.

(d) Compute the derivatives r_i of the matrices R_i at the identity matrix, that is, where the parameter is zero.

(e) Compute the matrix commutators $[r_i, r_j]$. *The commutator of matrices is defined simply by $[A, B] = AB - BA$.*

(f) Discuss your results.