HW #2

1. **Acceleration** (essentially a translation of problem 1.6.1 in B & G)
Let $\alpha : \mathbb{R} \longmapsto M$ be a curve. Recall that for any function $f : M \longmapsto \mathbb{R}$ on M, we have:

$$\left(\alpha_* \frac{d}{du}\right)(f) = \frac{d}{du}(f \circ \alpha)$$

The vector $\alpha_* \frac{d}{du} \in TM$ can be thought of as the *velocity* of the curve α.

(a) Suppose now that at a particular point $\alpha(0) = p \in M$ the velocity vanishes, that is

$$\alpha_* \frac{d}{du} = 0 \in T_pM$$

Show that the *acceleration* of the curve α, defined by

$$\left(\alpha_{**} \frac{d^2}{du^2}\right)(f) = \frac{d^2}{du^2}(f \circ \alpha)$$

is a tangent vector, that is, show that

$$\left(\alpha_{**} \frac{d^2}{du^2}\right) \in T_pM$$

(b) Show that the acceleration is *not* a tangent vector if the velocity is nonzero.

(c) Consider the parametric curve in $\mathbb{R}^2$ given by $x = u^2$, $y = u^4$. Use the above ideas to construct a nonzero tangent vector to this curve at the origin.