MTH 437/537 **HW #5** Spring 2024

1. **SHAPE OF SCHWARZSCHILD GEOMETRY**

(a) Plot the graph of $8mr = h^2 + 16m^2$ with r as the *horizontal* coordinate.
Equivalently, solve this equation for h, and plot h as a function of r.

(b) Express arclength along this graph in terms of r (and dr).
Assume that (r, h) are rectangular, Euclidean coordinates.

(c) Consider the surface of revolution obtained by rotating your curve about the h-axis. What is the line element for this surface? *Compare your answer with the Schwarzschild geometry.*

(d) Find the Gaussian curvature of this surface. Is it positive or negative?

2. **SCHWARZSCHILD CURVATURE**

(***Do not turn this problem in!*** *See me if you need help.*)

(a) Compute the Riemann curvature of the Schwarzschild geometry.

You may use any formalism you wish, and any coordinate system you wish. However, the results discussed in class will emphasize computations using differential forms in an orthonormal Schwarzschild ("shell") basis. Feel free to check your work using computer algebra, but make sure you understand the conventions being used! Your answer should consist of all independent nonzero curvature 2-forms (or components of the Riemann tensor). You can check your answer in many textbooks, including DFGGR, but again make sure you understand the conventions being used.

(b) Compute the components R_{ij} of the Ricci curvature of the Schwarzschild geometry.

A reasonable starting point would be the curvature 2-forms computed above, and the relations

$$\Omega^i j = \frac{1}{2} R^i{}_{jkl}\, \sigma^k \wedge \sigma^l$$
$$R_{ij} = R^m{}_{imj}$$

But again, you may use any formalism, and any coordinate system, and you should again feel free to check your answers using computer algebra. How many independent computations do you need to do? What answer do you expect?