MTH 437/537 **HW #3** Spring 2024

The line element for a Schwarzschild black hole takes the form:

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2m}{r}} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

All orbits can be assumed to lie in the equatorial plane ($\theta = \pi/2$).

1. **SATELLITE ORBITS**

(a) Find the speed of a satellite orbiting a Schwarzschild black hole at constant radius $r = 6m$, as measured by a stationary ("shell") observer at that radius.

(b) Is a circular orbit at $r = \frac{5}{2}m$ possible?

(c) Determine the smallest radius at which a circular orbit is possible, and the (shell) speed of a satellite in such an orbit

2. **NULL ORBITS**
Imagine a beam of light in orbit around a Schwarzschild black hole at constant radius.

(a) How fast would a shell observer think the beam of light is traveling?
Your answer must be supported by an explicit computation!

(b) How fast would an observer far away think the beam of light is traveling?
Recall that observers far away believe that t and r have their usual properties from special relativity. They are not really "observers" so much as "bookkeepers".

(c) At what value(s) of r, if any, is such an orbit possible?