You may wish to recall the following facts:

$$\cosh^2\beta - \sinh^2\beta = 1 \qquad \frac{v}{c} = \tanh\beta$$

$$\int \frac{1}{\sin\theta}\,d\theta = \ln\tan\frac{\theta}{2} \qquad \int \frac{a}{a^2-u^2}\,du = \operatorname{arctanh}\left(\frac{u}{a}\right)$$

$$d\hat{\boldsymbol{e}}_j = \omega^i{}_j\,\hat{\boldsymbol{e}}_i \qquad \omega_{ij} = \hat{\boldsymbol{e}}_i \cdot d\hat{\boldsymbol{e}}_j \qquad d\sigma^i + \omega^i{}_j \wedge \sigma^j = 0 \qquad \omega_{ij} + \omega_{ji} = 0$$

$$dx^2 - dt^2 = d\rho^2 - \rho^2 d\alpha^2 = -du\,dv \qquad x = \rho\cosh\alpha \qquad t = \rho\sinh\alpha$$

$$-\left(1-\frac{2m}{r}\right)dt^2 + \frac{dr^2}{1-\frac{2m}{r}} = -dT^2 + \left(dr + \sqrt{\frac{2m}{r}}\,dT\right)^2 = -\frac{32m^3}{r}e^{-r/2m}\,dU\,dV$$

$$\sigma^T = dT = dt + \frac{\sqrt{\frac{2m}{r}}}{1-\frac{2m}{r}}\,dr \qquad \sigma^R = \sqrt{\frac{2m}{r}}\,dR = \frac{dr}{1-\frac{2m}{r}} + \sqrt{\frac{2m}{r}}\,dt$$

$$ds^2 = d\vec{\boldsymbol{r}} \cdot d\vec{\boldsymbol{r}} \qquad \vec{\boldsymbol{v}} = \frac{d\vec{\boldsymbol{r}}}{d\lambda}$$

$$ds^2 = dr^2 + r^2 d\phi^2 \quad \Longleftrightarrow \quad \vec{\boldsymbol{v}} = \dot{r}\,\hat{\boldsymbol{r}} + r\dot{\phi}\,\hat{\boldsymbol{\phi}}$$

$$\text{Killing: } d\vec{\boldsymbol{X}} \cdot d\vec{\boldsymbol{r}} = 0 \qquad \text{geodesic: } d\vec{\boldsymbol{v}} = 0$$

$$\text{Schwarzschild:} \quad \dot{\phi} = \frac{\ell}{r^2} \qquad \dot{t} = e \Big/ \left(1-\frac{2m}{r}\right)$$

$$\dot{r}^2 = \begin{cases} e^2 - \left(1+\frac{\ell^2}{r^2}\right)\left(1-\frac{2m}{r}\right) & \text{(timelike)} \\ e^2 - \left(1-\frac{2m}{r}\right)\frac{\ell^2}{r^2} & \text{(null)} \end{cases}$$