MTH 437/537

## HW #3

Spring 2021

The line element for a Schwarzschild black hole takes the form:

$$ds^{2} = -\left(1 - \frac{2m}{r}\right)dt^{2} + \frac{dr^{2}}{1 - \frac{2m}{r}} + r^{2}\left(d\theta^{2} + \sin^{2}\theta \, d\phi^{2}\right)$$

All orbits can be assumed to lie in the equatorial plane  $(\theta = \pi/2)$ .

## 1. SATELLITE ORBITS

- (a) Find the speed of a satellite orbiting a Schwarzschild black hole at constant radius r = 6m, as measured by a stationary ("shell") observer at that radius.
- (b) Is a circular orbit at  $r = \frac{5}{2}m$  possible?
- (c) Determine the smallest radius at which a circular orbit is possible, and the (shell) speed of a satellite in such an orbit

## 2. NULL ORBITS

Imagine a beam of light in orbit around a Schwarzschild black hole at constant radius.

- (a) How fast would a shell observer think the beam of light is traveling? Your answer must be supported by an explicit computation!
- (b) How fast would an observer far away think the beam of light is traveling? Recall that observers far away believe that t and r have their usual properties from special relativity. They are not really "observers" so much as "bookkeepers".
- (c) At what value(s) of r, if any, is such an orbit possible?