

HOUSE RULES

*This is a closed-book exam. You may not use any resources of any kind other than writing implements. You may typeset your answers if you wish, but this is not required. The use of other online or local resources, such as Mathematica or Wikipedia, is not permitted. You may **not** discuss this exam with anyone other than Tevian Dray. This exam may not be shared publicly in any form, such as online. Your signature above acknowledges that you have complied with these rules.*

To reach me during the exam, use email and/or the Office Hours Zoom link.

You may wish to recall the following facts:

$$\begin{aligned} \cosh^2 \beta - \sinh^2 \beta &= 1 & \frac{v}{c} &= \tanh \beta \\ \int \frac{1}{\sin \theta} d\theta &= \ln \tan \frac{\theta}{2} & \int \frac{a}{a^2 - u^2} du &= \operatorname{arctanh} \left(\frac{u}{a} \right) \\ dx^2 - dt^2 &= d\rho^2 - \rho^2 d\alpha^2 = -du dv & x &= \rho \cosh \alpha \quad t = \rho \sinh \alpha \\ - \left(1 - \frac{2m}{r} \right) dt^2 + \frac{dr^2}{1 - \frac{2m}{r}} &= -dT^2 + \left(dr + \sqrt{\frac{2m}{r}} dT \right)^2 = -\frac{32m^3}{r} e^{-r/2m} dU dV \\ \sigma^T = dT &= dt + \frac{\sqrt{\frac{2m}{r}}}{1 - \frac{2m}{r}} dr & \sigma^R &= \sqrt{\frac{2m}{r}} dR = \frac{dr}{1 - \frac{2m}{r}} + \sqrt{\frac{2m}{r}} dt \\ \text{geodesic: } d\vec{v} &= 0 & \text{Killing: } d\vec{X} \cdot d\vec{r} &= 0 \\ \dot{\phi} &= \frac{\ell}{r^2} & \dot{t} &= e \left/ \left(1 - \frac{2m}{r} \right) \right. \\ \dot{r}^2 &= \begin{cases} e^2 - \left(1 + \frac{\ell^2}{r^2} \right) \left(1 - \frac{2m}{r} \right) & \text{(timelike)} \\ e^2 - \left(1 - \frac{2m}{r} \right) \frac{\ell^2}{r^2} & \text{(null)} \end{cases} \end{aligned}$$