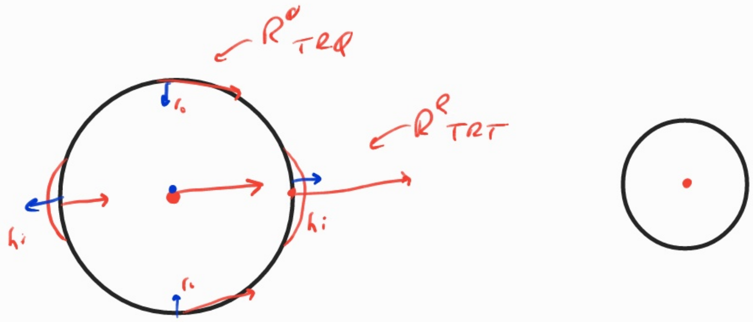
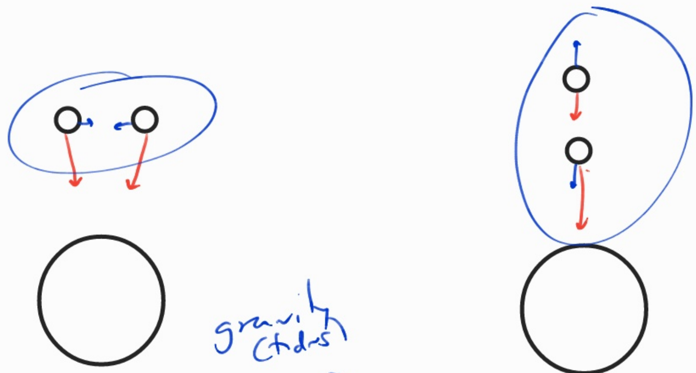
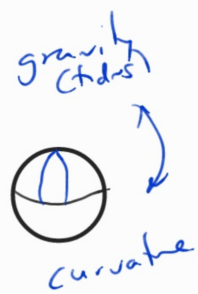






sphere

Curvature 2-forms (Schw − rain)

$$\Omega^{\Phi}{}_{T} = \Omega^{T}{}_{\Phi} = -\frac{m}{r^3}\,\sigma^T \wedge \sigma^\Phi$$

$$\Omega^{T}{}_{R} = \frac{2m}{r^3}\,\sigma^T \wedge \sigma^R$$

① $\Omega^{\Phi}{}_{T} = \Omega^{T}{}_{\Phi}$

$\Omega^{R}{}_{T} = \Omega^{T}{}_{R}$

$$\boxed{\begin{aligned} \omega_{ij} &= -\omega_{ji} \\ \Omega_{ij} &= -\Omega_{ji} \end{aligned}}$$

② $\Omega^{i}{}_{j} = \frac{1}{2} R^{i}{}_{jkl}\,\sigma^k \wedge \sigma^l$

$$\Omega^{\Phi}{}_{T} = \frac{1}{2} R^{\Phi}{}_{T\Phi T}\,\sigma^\Phi \wedge \sigma^T + \frac{1}{2} R^{\Phi}{}_{TT\Phi}\,\sigma^T \wedge \sigma^\Phi + \cancel{\ldots}$$

$$= -\frac{m}{r^3}\,\sigma^T \wedge \sigma^\Phi$$

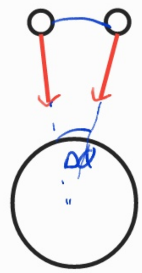
$$\Rightarrow \boxed{R^{\Phi}{}_{T\Phi T} = +\frac{m}{r^3}}$$

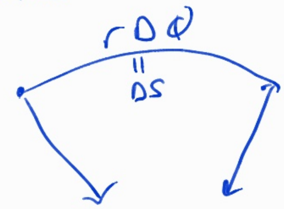
$$\boxed{R^{R}{}_{TRT} = -\frac{2m}{r^3}}$$

geodesic separation

general: $\ddot{\Delta S} = -R^{x}{}_{TxT}\,\Delta S$

nearby radial geodesics





Q: Find $(r\Delta\phi)^{\cdot\cdot}$ ← w.r.t. τ

$\left(f = 1 - \frac{2m}{r}\right)$

$f' = \frac{2m}{r^2}$

Know: $\dot r^2 = e^2 - f$

$\dot\phi = 0 \Rightarrow (\Delta\phi)^{\cdot} = 0$

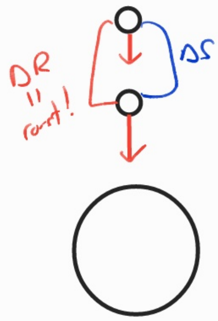
$$(\Delta s)^{\overset{\tau}{\cdot\cdot}} = (r\Delta\phi)^{\cdot\cdot} = \ddot r\,\Delta\phi = -\frac{m}{r^2}\Delta\phi = -\frac{m}{r^3}\Delta s$$

ϕ ϕ

$$2\cancel{\dot r}\ddot r = 0 - f'\cancel{\dot r}$$

$$\Rightarrow \ddot r = -\frac{1}{2}f' = -\frac{m}{r^2}$$

$R = \text{const}!$
(for rain obs)



$$ds^2 = -dT^2 + \underline{\frac{2m}{r}\,dR^2}$$

$$\Delta S = \sqrt{\frac{2m}{r}}\,\Delta R$$

$\underline{\text{want}}$ $(\Delta S)^{\cdot\cdot} = \left(\sqrt{\frac{2m}{r}}\right)^{\cdot\cdot} \Delta R$

$$\dot{r}^2 = 1 - f = \frac{2m}{r} \Rightarrow \dot{r} = -\sqrt{\frac{2m}{r}}$$

$$\ddot{r} = -\frac{m}{r^2}$$

$$\Delta S = -\dot{r}\,\Delta R$$

$$(\Delta S)^{\cdot} = -\ddot{r}\,\Delta R = +\frac{m}{r^2}\,\Delta R$$

$$(\Delta S)^{\cdot\cdot} = -\frac{2m}{r^3}\,\dot{r}\,\Delta R = +\frac{2m}{r^3}\,\Delta S$$