MTH 437/537 **HW #6** Spring 2017

0. **WARMUP** *(Do not turn this problem in, but see me if you need help.)*
Determine the (nonzero) components $R^i{}_{jkl}$ of the curvature 2-forms

$$\Omega^i j = \frac{1}{2} R^i{}_{jkl}\, \sigma^k \wedge \sigma^l$$

for the Robertson-Walker geometry, with line element

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2 \right)$$

with $k = -1, 0, 1$ depending on whether the spatial cross-sections are hyperbolic, flat, or spherical, respectively.

1. The components of the Einstein vector-valued 1-form $\vec{\boldsymbol{G}} = G^i{}_j \sigma^j \hat{\boldsymbol{e}}_i$ are related to the components of the Ricci vector-valued 1-form $\vec{\boldsymbol{R}} = R^i{}_j \sigma^j \hat{\boldsymbol{e}}_i$ by the relationship

$$G^i{}_j = R^i{}_j - \frac{1}{2}\, \delta^i{}_j\, R.$$

Find an expression for the "Einstein scalar" $G = G^i{}_i$ in terms of the Ricci scalar $R = R^i{}_i$.
Do not assume the geometry is given; this question is not about any particular spacetime.

2. Compute the (nonzero) components $G^i{}_j$ of the Einstein vector-valued 1-form $\vec{\boldsymbol{G}}$ for the Robertson-Walker geometry.