MTH 437/537

HW #6

0. **WARMUP** (Do not turn this problem in, but see me if you need help.) Using the relation

$$\Omega^i j = \frac{1}{2} R^i{}_{jkl} \, \sigma^k \wedge \sigma^l$$

or otherwise, compute the (nonzero) components $R^i{}_{jkl}$ of the Riemann curvature of the Robertson-Walker geometry, with line element

$$ds^{2} = -dt^{2} + a(t)^{2} \left(\frac{dr^{2}}{1 - kr^{2}} + r^{2} d\theta^{2} + r^{2} \sin^{2} \theta d\phi^{2} \right)$$

with k = -1, 0, 1 depending on whether the spatial cross-sections are hyperbolic, flat, or spherical, respectively.

1. Using the relationships

$$R_{ij} = R^m{}_{imj}$$
$$G^i{}_j = R^i{}_j - \frac{1}{2}\,\delta^i{}_j\,R$$

compute the (nonzero) components $G^i{}_j$ of the Einstein tensor for the Robertson-Walker geometry.