MTH 437/537 **HW #6** Spring 2013

1. Compute the *curvature* of the Robertson-Walker line element

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2 \right)$$

with $k = -1, 0, 1$ depending on whether the spatial cross-sections are hyperbolic, flat, or spherical, respectively.

You may use any method you wish. It is sufficient to determine either the nonzero curvature 2-forms, or the nonzero components of the Riemann tensor, in any basis (not necessarily orthonormal) and index structure (combination of up and down indices), but in both cases it must be clear which one you are using. A complete (and correct) but otherwise undocumented computer printout will receive full credit.

2.

(a) Using the relationship

$$G^i{}_j = R^i{}_j - \frac{1}{2}\,\delta^i{}_j\, R$$

find an expression for the "Einstein scalar" $G = G^i{}_i$ in terms of the Ricci scalar $R = R^i{}_i$.

(b) Determine the Ricci scalar for the spacetime given in the previous problem.

(c) Can a vacuum solution of Einstein's equation (with zero cosmological constant) have $R \neq 0$?

MTH 437/537 **HW #7** Spring 2013

Compute the *Einstein tensor* for the Robertson-Walker line element

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2 \right)$$

with $k = -1, 0, 1$ depending on whether the spatial cross-sections are hyperbolic, flat, or spherical, respectively.

You may use any method you wish. It is sufficient to determine either the nonzero components of the Einstein tensor or the (nonzero) Einstein 1-forms (or 3-forms) in any basis and index structure. A complete (and correct) but otherwise undocumented computer printout will receive full credit. You are strongly encouraged to reuse your (correct!) computation from the previous assignment.