HW #2

The line element for a Schwarzschild black hole takes the form::

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2m}{r}} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

where the mass m and the radius r are measured in the same units.

1. **PROPER DISTANCE BETWEEN SHELLS**

(a) Use the line element to find a formula for the proper distance between *nearby* spherical shells (surfaces with $r =$ constant). That is, find an expression for the infinitesimal distance between nearby shells, assuming that only the radius changes (and that $r > 2m$).

(b) As you approach the horizon ($r \to 2m^+$), what happens to your expression? How far away do you think the horizon is? Do you think that you can ever get to the horizon?

2. **APPROXIMATE PROPER DISTANCE**

The mass m of a particular black hole is 5 km, a little more than three times that of our Sun. Two concentric spherical shells surround this black hole. The inner shell has circumference $2\pi r$, and the outer shell has circumference $2\pi(r + \Delta r)$, where Δr is 100 cm. Use your expression for the infinitesimal distance between nearby shells to estimate the radial distance between the shells in each of the cases below. *Explicitly state any approximations you make.*

(a) $r = 50$ km

(b) $r = 15$ km

(c) $r = 10.5$ km

3. **EXACT PROPER DISTANCE**

(a) Use your expression for the infinitesimal distance between nearby shells to determine the exact (radial) distance traveled between two spherical shells of arbitrary circumference (but outside the horizon, that is, with $r > 2m$).

(b) Use your result to decide whether the radial distance to the horizon is finite or infinite.

You should feel free to use integral tables or computer algebra, but should document doing so. If you prefer to integrate by hand, the substitution $\cosh\alpha = \sqrt{\frac{r}{2m}}$ *may be helpful. Finally, you may want to use your result to check the accuracy of your earlier approximations.*