

### 1. VECTORS IN MINKOWSKI SPACE

Show that a timelike vector cannot be orthogonal to a null vector or to another timelike vector. Show that two null vectors are orthogonal if and only if they are parallel. (Assume these vectors are nonzero.)

*Try to do this in 4 spacetime dimensions, rather than 2. A convenient notation is to view a 4-vector  $\mathbf{u}$  as consisting of a timelike component  $u^t$  and spacelike components making up an ordinary 3-vector  $\vec{\mathbf{u}}$ ; one often writes*

$$\mathbf{u} = \begin{pmatrix} u^t \\ \vec{\mathbf{u}} \end{pmatrix}$$

### 2. EARTH DISTANCE

Corvallis is located at approximately (44.6°N,123.3°W), that is, 44.6° north of the equator (latitude), and 123.3° west of the prime meridian (longitude). Tangent is located at approximately (44.6°N,123.1°W), and Eugene, is at approximately (44.1°N, 123.1°W).

Portland is located at approximately (45.5°N,122.5°W), that is, 45.5° north of the equator (latitude), and 122.5° west of the prime meridian (longitude). San Francisco is located at approximately (37.5°N,122.5°W), and Richmond, VA, is at approximately (37.5°N, 77.5°W).

*Assume the Earth is a perfect sphere, with radius  $r \approx 4000$  miles, and the line element:*

$$ds^2 = r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

- Find the approximate distance between Corvallis and Tangent.
- Find the approximate distance between Tangent and Eugene.
- Find the approximate distance between Corvallis and Eugene.
- Find the approximate distance between Portland and San Francisco.
- Find the approximate distance between San Francisco and Richmond.
- Find the approximate distance between Portland and Richmond.
- How good are your approximations?

*You should avoid doing messy computations! If you really feel one is necessary, it is sufficient to describe the computation without completing it.*

### 3. THE GETAWAY

*This problem is optional, but good practice. Do not turn it in, but feel free to discuss it with me. Do try to solve it without help first.*

The outlaws are escaping in their getaway car, which goes  $\frac{3}{4}c$ , chased by the police, moving at only  $\frac{1}{2}c$ . Realizing they can't catch up, the police attempt to shoot out the tires of the getaway car. Their guns have a muzzle velocity (speed of the bullets relative to the gun) of  $\frac{1}{3}c$ .

- Does the bullet reach its target according to Galileo?
- Does the bullet reach its target according to Einstein?
- Verify that your answer to part (b) is the same in all four (!) reference frames: ground, police, outlaws, and bullet.