MTH 437/537 **FINAL EXAM COMMENTS** Spring 2013

1. The metric coefficients are functions of one variable! Rewriting them as arbitrary functions of two variables just makes extra work. Explicitly, if $f = f(u)$ and $u = t - x$, then

$$df = f'(u)\,du = f'(u)(dt - dx)$$

Put differently,

$$\frac{\partial f}{\partial t} = \frac{df}{du}\frac{\partial u}{\partial t} = \frac{df}{du}$$
$$\frac{\partial f}{\partial x} = \frac{df}{du}\frac{\partial u}{\partial x} = -\frac{df}{du}$$

Also, don't forget that the Ricci and Einstein tensors have 16 components! They are symmetric, so that only 10 components are independent. But it is *not* sufficient to consider only the four diagonal components.

2. Null geodesics are different from timelike geodesics... (It is possible to analyze the behavior of null geodesics is to treat them as a limiting case of timelike geodesics, but this approach requires doing some of problem 4 first.)

3. Roberstson-Walker cosmological models with $k = +1$ have spatial cross-sections that are *3-spheres*. What are the coordinates on these spheres? (ψ,θ,ϕ). Since $r = \sin\psi$, r is also a coordinate *on* the 3-sphere; it is *not* the radius of the 3-sphere. (What is the radius? That's a.) A beam of light moving in the "radial" (i.e. r) direction moves around the 3-sphere; the question is whether it makes it all the way around during the lifetime of the universe.

   A good way to visualize a 3-sphere is to suppress θ, as we often do in the presence of spherical symmetry, by setting $\theta = \pi/2$. The 3-sphere metric now reduces to a 2-sphere metric, describing the "equator" of the 3-sphere.

4. If an object has zero velocity, but nonzero acceleration, it does *not* come to rest. What does it do instead? Yes, this really happens for these falling objects!