MTH 437/537 **HW #7** Spring 2011

1. Compute the Einstein tensor for the Robertson-Walker line element

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 \, d\theta^2 + r^2 \sin^2 \theta \, d\phi^2 \right)$$

with $k = -1, 0, 1$ depending on whether the spatial cross-sections are hyperbolic, flat, or spherical, respectively.

You may use any method you wish. It is sufficient to determine the nonzero components of the Einstein tensor in any basis and index structure (combination of up and down indices), but in both cases it must be clear which one you are using. Alternatively, you may determine the (nonzero) Einstein 1-forms (or 3-forms). A complete (and correct) but otherwise undocumented computer printout will receive full credit.

2.

(a) Using the relationship

$$G^i{}_j = R^i{}_j - \frac{1}{2} \delta^i{}_j \, R$$

find an expression for the "Einstein scalar" $G = G^i{}_i$ in terms of the Ricci scalar $R = R^i{}_i$.

(b) Determine the Ricci scalar for the spacetime given in the previous problem.

(c) Can a vacuum solution of Einstein's equation (with zero cosmological constant) have $R \neq 0$?