MTH 437/537 **HW #3** Spring 2011

All references to black holes assume the Schwarzschild line element:

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2m}{r}} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

1. **PROPER DISTANCE BETWEEN SHELLS**

The mass m of a black hole is 5 km, a little more than three times that of our Sun. Two concentric spherical shells surround this black hole. The inner shell has circumference $2\pi r$, and the outer shell has circumference $2\pi(r + \Delta r)$, where Δr is 100 cm. Estimate the radial distance between the shells in each of the cases below.
Explicitly state any approximations you make.

(a) $r = 50$ km

(b) $r = 15$ km

(c) $r = 10.5$ km

2. **ZENO'S PARADOX**

The radial part ($dt = d\theta = d\phi = 0$) of the Schwarzschild line element is

$$ds^2 = \frac{dr^2}{1 - \frac{2m}{r}}$$

As you approach the horizon ($r \to 2m^+$), the radial distance you travel must therefore approach ∞. Zeno therefore asserts, an object dropped into (towards) a black hole can never reach the horizon.

(a) Determine the (radial) distance traveled between two spherical shells of arbitrary circumference (but outside the horizon, that is, with $r > 2m$).

(b) Use your result to decide whether the radial distance to the horizon is finite or infinite.
You should feel free to use integral tables or computer algebra, but should document doing so. If you prefer to integrate by hand, the substitution $\cosh\alpha = \sqrt{\frac{r}{2m}}$ *may be helpful. Finally, you may want to use your result to check the accuracy of your earlier approximations.*