MTH 437/537 **HW #6** Spring 2007

The general Robertson-Walker (isotropic, homogeneous) cosmological model has line element

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2 \right)$$

with $k = -1, 0, 1$ depending on whether the spatial cross-sections are hyperbolic, flat, or spherical, respectively. The only nonzero components of the Einstein tensor are

$$G^t{}_t = -3\,\frac{\dot{a}^2 + k}{a^2} \quad ; \quad G^r{}_r = G^\theta{}_\theta = G^\phi{}_\phi = -\,\frac{2a\ddot{a} + \dot{a}^2 + k}{a^2}$$

(You do not need to verify this.)

1. **CURVATURE SCALARS**

The Einstein tensor is related to the Ricci tensor by

$$G^i{}_j = R^i{}_j - \frac{1}{2}\,\delta^i{}_j\, R$$

(a) Find an expression for the "Einstein scalar" $G = G^i{}_i$ in terms of the Ricci scalar $R = R^i{}_i$.

(b) Determine the Ricci scalar for the Robertson-Walker model above.

(c) ***(Optional!)*** Derive at least one component of the Einstein tensor above for the case $k = 0$.