

### 1. FALLING TOWARDS A BLACK HOLE

Consider a falling object which starts at rest with respect to the spherical shell  $r = r_0$ .

- (a) According to a faraway observer, how fast does the object appear to be falling as it crosses a smaller shell whose circumference is  $2\pi r$ ?
- (b) According to an observer on that smaller shell, how fast does the object appear to be falling as it goes past?
- (c) Find the limiting values of these speeds as the object approaches the horizon.

*What, if anything, do you think these results mean physically? You may want to think about what happens “at” the horizon. For instance, what is the limiting value of  $\frac{dt}{d\tau}$ ? If you “stand still” “on” the horizon, what sort of trajectory are you following? How fast does this mean you are going?*

### 2. NO ESCAPE FROM A BLACK HOLE

The goal of this problem is to establish an upper bound on the maximum proper time it takes to reach the singularity at  $r = 0$  starting at the horizon  $r = 2m$ .

- (a) Use the line element to show that *inside* the horizon the following relation holds:

$$\left| \frac{dr}{d\tau} \right| \geq \sqrt{\frac{2m}{r} - 1}$$

*Assume that the Schwarzschild line element is valid inside the horizon, that is, for  $r < 2m$ , and recall that the proper time along any timelike trajectory is given by  $d\tau^2 = -ds^2$ .*

- (b) Assuming equality, find the elapsed proper time to travel from  $r = 2m$  to  $r = 0$ ; this is the desired upper bound.

*You should feel free to use integral tables or computer algebra, but should document doing so. If you prefer to integrate by hand, the substitution  $\sin \alpha = \sqrt{\frac{r}{2m}}$  may be helpful, and you may wish to recall that  $\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$ .*

- (c) How long does it take to reach the center of a solar mass black hole, starting at the horizon, if you do everything you can to resist falling in?