

All references to black holes assume the Schwarzschild line element:

$$ds^2 = - \left(1 - \frac{2m}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2m}{r}} + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

1. PROPER DISTANCE BETWEEN SHELLS

The mass m of a black hole is 5 km, a little more than three times that of our Sun. Two concentric spherical shells surround this black hole. The inner shell has circumference $2\pi r$, and the outer shell has circumference $2\pi(r + \Delta r)$, where Δr is 100 cm. Determine the radial distance between the shells in each of the cases below.

Approximate answers accurate to the nearest centimeter are acceptable.

- (a) $r = 50$ km
- (b) $r = 15$ km
- (c) $r = 10.5$ km

2. HOW MANY SHELLS?

The Schwarzschild radius of Black Hole Alpha is 10 km. Your construction company agrees to build 1000 spherical shells around Black Hole Alpha with circumferences $2\pi(10.001)$ km, $2\pi(10.002)$ km, ..., $2\pi(11.000)$ km. You determine the location of the next shell by mounting a 1-meter rod vertically (radially) on each completed shell, and you are careful to verify the length of the rod in place before starting work on the new shell at the end of the rod.

- (a) What is the radial distance between your shells?
- (b) Is the difference in circumference between adjacent shells 200π cm, as called for in your contract?
- (c) What should the radial distance be between the two innermost shells?
If you use an approximation, you should discuss its accuracy.

3. ZENO'S PARADOX

The radial part ($dt = d\theta = d\phi = 0$) of the Schwarzschild line element is

$$ds^2 = \frac{dr^2}{1 - \frac{2m}{r}}$$

As you approach the horizon ($r \rightarrow 2m^+$), the radial distance you travel must therefore approach ∞ . Zeno therefore asserts, an object dropped into (towards) a black hole can never reach the horizon.

- (a) Determine the (radial) distance traveled between two spherical shells of arbitrary circumference (but outside the horizon, that is, with $r > 2m$).
- (b) Use your result to decide whether the radial distance to the horizon is finite or infinite.
You should feel free to use integral tables or computer algebra, but should document doing so. If you prefer to integrate by hand, the substitution $\cosh \alpha = \sqrt{\frac{r}{2m}}$ may be helpful. Finally, you may want to use your result to check the accuracy of your earlier approximations.