

1. **GLOBAL STRUCTURE**

Determine the two families of null curves in the (T, X) plane (i.e. with Y and Z constant) for the metric

$$ds^2 = -dT^2 - 2 \sinh(X) dT dX + dX^2 + dY^2 + dZ^2$$

and express T as a function of X in each case. Define coordinates U, V in terms of X, T so that on each such null curve either $U=\text{constant}$ or $V=\text{constant}$. By first transforming to coordinates (U, V, Y, Z) , or otherwise, find a coordinate transformation which shows that this metric is just the Minkowski metric

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

HINT: The Minkowski metric in double-null coordinates takes the form

$$ds^2 = -du dv + dy^2 + dz^2$$

where $u = t - x$ and $v = t + x$.