MTH 437/537 **HW #3** Spring 1999

1. **CURVATURE OF THE SPHERE**

Consider the sphere with radius r, whose line element is

$$ds^2 = r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2$$

Do not worry about the fact that these coordinates do not cover all of the sphere.

(a) Find all nonzero Christoffel symbols $\Gamma^i{}_{jk}$

(b) Find all nonzero components of the Riemann tensor ($R^i{}_{jkl}$) and the Ricci tensor (R_{ij}), and calculate the Ricci scalar R.

Use the symmetry properties of the Riemann tensor to avoid unnecessary computation. How many independent components are there?

GROUND RULES:

You may use any basis you wish; the 2 standard choices are a coordinate basis, in which case you should probably use the formulas for the components of these tensors, and an orthonormal basis, in which case you may prefer to work with differential forms.

*You **may** use algebraic software to check your answer, but then please include a printout of your session. You still must show me your full calculation by hand.*

2. **GEODESICS ON THE SPHERE**

This problem is straightforward once you have found the Christoffel symbols above.

(a) Write down, but **do not solve**, the differential equations for geodesics on the sphere.

*It is possible to find the general solution of these equations in closed form. A possibly subtle integration is involved; be warned that some versions of algebraic software get this integral wrong! You are **not** expected to attempt this, but by all means see me if you are interested.*

(b) Verify that the equator is a geodesic.

(c) Argue on the basis of symmetry that the geodesics on the sphere are the great circles.