MTH 437/537 **HW #2** Spring 1999

1. **VECTORS IN MINKOWSKI SPACE** (d'Inverno exercise 8.2 on page 119)

Show that (in Minkowski space) a timelike vector cannot be orthogonal to a null vector or to another timelike vector. Show that two null vectors are orthogonal if and only if they are parallel.

HINT: You may want to write 4-vectors as

$$(u^i) = \begin{pmatrix} u^0 \\ \vec{u} \end{pmatrix}$$

so that the inner product can be written

$$g(u, v) = g_{ij} u^i v^j = \vec{u} \cdot \vec{v} - u^0 v^0$$

where "·" denotes the usual 3-dimensional dot product. Now use $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos \theta$.

WARNING: I use the opposite sign convention from d'Inverno, so that my metric corresponds to the line element

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

and I define vectors to be timelike if their norm ("squared") is **negative** *and spacelike if their norm ("squared") is* **positive**.

2. **NONDIAGONAL METRICS**

Consider the coordinates $u = x - y$, $v = y$ in $\mathbb{R}^2$ with the usual line element (metric) $ds^2 = dx^2 + dy^2$.

(a) Express the coordinate basis vectors $\frac{\partial}{\partial u}$, $\frac{\partial}{\partial v}$ and the corresponding dual basis 1-forms du, dv in terms of the coordinates (x, y).

WARNING: Don't make assumptions!

(b) Determine the metric (line element) in coordinates (u, v), that is, in terms of du and dv.