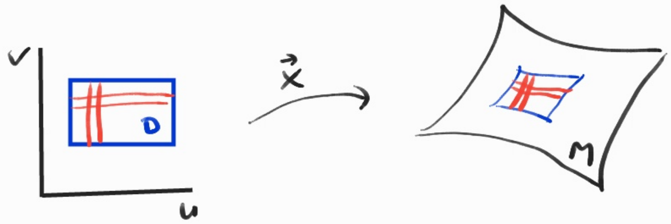
Recall:



- tangent vectors:

$$\vec{e}_1 = \frac{\partial \vec{x}}{\partial u}, \quad \vec{e}_2 = \frac{\partial \vec{x}}{\partial v} \quad \leftarrow \text{coordinate basis}$$

$$\vec{v} \in TM \Leftrightarrow \vec{v} = v_i \vec{e}_i$$

- dual basis: (for any basis)

$$\sigma_i(\vec{e}_j) = \delta_{ij}$$

- 0-forms:

$$f : M \to \mathbb{R}$$

- 1-forms:

$$\phi : TM \to \mathbb{R} \quad \underline{\text{linear}}$$

$$\Rightarrow \phi = \phi_i \sigma_i$$

- 2-forms:

$$\eta : TM \times TM \to \mathbb{R} \quad \underline{\text{bilinear}}, \ \underline{\text{antisymmetric}}$$

$$\Rightarrow \eta = h\, \sigma_1 \wedge \sigma_2$$

$\wedge$ where $\boxed{\phi \wedge \psi(\vec{v}, \vec{w}) = \begin{vmatrix} \phi(\vec{v}) & \phi(\vec{w}) \\ \psi(\vec{v}) & \psi(\vec{w}) \end{vmatrix}}$ $\Rightarrow du \wedge dv(\vec{e}_1, \vec{e}_2) = 1$

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d $\quad \boxed{dg(\vec{v}) = \vec{v}[g]}$

$$\vec{e}_1[f] = \frac{\partial f}{\partial u} \Rightarrow du(\vec{e}_1) = \vec{e}_1[u] = 1$$

$$dv(\vec{e}_1) = \vec{e}_1[v] = 0$$

$\Rightarrow$ dual basis is $\{du, dv\}$

$$\Rightarrow \phi = \phi_1 du + \phi_2 dv$$

$$\eta = h\, du \wedge dv$$

$$\Rightarrow d\phi = d\phi_1 \wedge du + d\phi_2 \wedge dv$$

$$= \left(\frac{\partial \phi_2}{\partial u} - \frac{\partial \phi_1}{\partial v}\right) du \wedge dv$$

$$\Rightarrow d\phi(\vec{e}_1, \vec{e}_2) = \frac{\partial \phi_2}{\partial u} - \frac{\partial \phi_1}{\partial v} = \frac{\partial}{\partial u}\phi(\vec{e}_2) - \frac{\partial}{\partial v}\phi(\vec{e}_1)$$