Stokes' Theorem

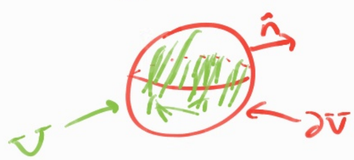
$$\boxed{\int_{\partial R} \beta = \int_R d\beta}$$

any rank
any manifold

Boundary

"∂"

$\partial I = b - a$ $\quad$ $\partial \alpha = B - A$



$$\Rightarrow \int_{I=[a,b]} df = \int_{\partial I = b-a} f =: f\Big|_a^b$$

$$\int_\alpha df = \int_{\partial\alpha} f = f\Big|_A^B$$

$$\int_X dF = \int_{\partial X} F \qquad \begin{aligned} F &= \vec{F}\cdot d\vec{r} \\ dF &= \vec{\nabla}\times\vec{F}\cdot d\vec{A} \end{aligned}$$

$$\therefore \int_R \vec{\nabla}\times\vec{F}\cdot d\vec{A} = \int_{\partial R} \vec{F}\cdot d\vec{r}$$

$$\int_V d{*}F = \int_{\partial V} {*}F \qquad \begin{aligned} d{*}F &= \vec{\nabla}\cdot\vec{F}\,dV \\ {*}F &= \vec{F}\cdot d\vec{A} \end{aligned}$$

$$\therefore \int_V \vec{\nabla}\cdot\vec{F}\,dV = \int_{\partial V} \vec{F}\cdot d\vec{A}$$

Examples

① $\Phi = x\,dy \Rightarrow d\Phi = dx\wedge dy$

$$\therefore \int_R d\Phi = \int_{\partial R} \Phi$$

$$\int_R d\Phi = \int_R dx\wedge dy, \qquad \int_{\partial R} \Phi = \oint_{\partial R} x\,d\Phi$$

$$\int_R dx\wedge dy = \iint dx\,dy$$

$$\text{Area} = \iint dx\,dy$$

② $\eta = z\,dx\wedge dy \Rightarrow d\eta = dz\wedge dx\wedge dy$

$$= +\,dx\wedge dy\wedge dz$$

$$\Rightarrow \int_V d\eta = \int_{\partial V} \eta$$

$$\int_V d\eta = \int_V dx\wedge dy\wedge dz, \qquad \int_{\partial V} \eta = \int_{\partial V} z\,dx\wedge dy$$

$$\int_V dx\wedge dy\wedge dz = \iiint dx\,dy\,dz, \qquad \int_{\partial V} z\,dx\wedge dy = \iint z\,dx\,dy$$

$$\text{Volume} = \iiint dx\,dy\,dz$$