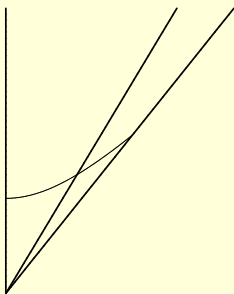


Drawing Spacetime Diagrams

- Points in spacetime are called *events*.
- Lines with slope $m = \pm 1$ represent beams of light.
- Vertical lines represent the worldline of an object at rest.
- Horizontal lines represent snapshots of constant time, that is, events which are simultaneous (in the given reference frame).
- Lines with slope $|m| > 1$ (called *timelike*) represent the worldlines of observers moving at constant speed.
- The speed of such an observer is given by $c \tanh \beta$, where β is the (hyperbolic) angle between the worldline and a *vertical* line.
- The distance between two events along such a line is just the time between them as measured by the moving observer.
- Lines with slope $|m| < 1$ (called *spacelike*) represent lines of simultaneity as seen by an observer moving at constant speed.
- The distance between two events along such a line is just the distance between them as measured by the corresponding observer.

Addition of Velocities



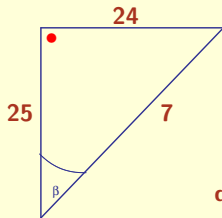
$$\frac{v}{c} = \tanh \beta$$

$$\tanh(\alpha + \beta) = \frac{\tanh \alpha + \tanh \beta}{1 + \tanh \alpha \tanh \beta} = \frac{\frac{u}{c} + \frac{v}{c}}{1 + \frac{uv}{c^2}}$$

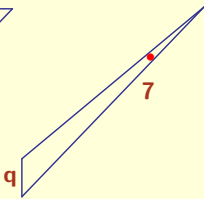
Einstein addition formula!

Twin Paradox

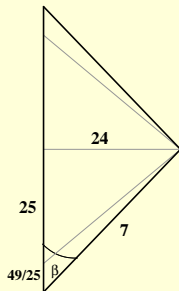
One twin travels 24 light-years to star X at speed $\frac{24}{25}c$; her twin brother stays home. When the traveling twin gets to star X, she immediately turns around, and returns at the same speed. How long does each twin think the trip took?



$$\cosh \beta = \frac{25}{7}$$



$$q = \frac{7}{\cosh \beta} = \frac{49}{25}$$



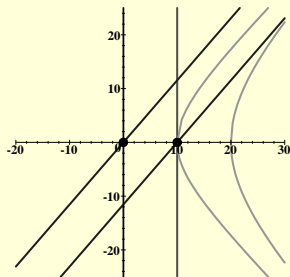
Straight path takes longest!

The Pole and the Barn

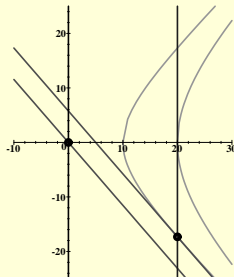
A 20 foot pole is moving towards a 10 foot barn fast enough that the pole appears to be only 10 feet long. As soon as both ends of the pole are in the barn, slam the doors. How can a 20 foot pole fit into a 10 foot barn? Draw a spacetime diagram!

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BARN



POLE