Vector-Valued Forms

$ds^2 = d\vec{r} \cdot d\vec{r}$

$d\vec{r} = \sigma_i \hat{e}_i$

- n-dimensional
- $\hat{e}_i \cdot \hat{e}_j = \pm \delta_{ij}$

signature
$s = \#$ of $-$

connection

$d\hat{e}_j = \omega_{ij} \hat{e}_i$

$\omega_{ij} = \hat{e}_i \cdot d\hat{e}_j$

structure equations

$d\sigma_i + \omega_{ij} \wedge \sigma^j = 0$

$\omega_{ij} + \omega_{ji} = 0$

curvature

$d^2 \hat{e}_j = \Omega_{ij} \hat{e}_i$

$\Rightarrow \Omega_{ij} = d\omega_{ij} + \omega_{ik} \wedge \omega_{kj}$

components

$\omega_{ij} = \Gamma_{ijk} \sigma^k$

$\Omega_{ij} = \frac{1}{2} R_{ijkl} \sigma_k \wedge \sigma_l$

Structure Equations

$$\Theta_i = d\sigma_i + \omega_{ij} \wedge \sigma_j$$ torsion

$$\Omega_{ij} = d\omega_{ij} + \omega_{ik} \wedge \omega_{kj}$$ curvature

$$d^2\vec{r} = \Theta_i \hat{e}_i$$

$$d^2\hat{e}_j = \Omega_{ij} \hat{e}_i$$

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orthonormal $\Rightarrow \omega_{ij} + \omega_{ji} = 0$

Relativity

	$s=0$	$s=1$
flat	Euclidean $\mathbb{R}^n$	Minkowskian SR
curved	Riemannian S^n	Lorentzian GR