Pullback

$F: M \to N$

$\vec{v} \mapsto F_* v$

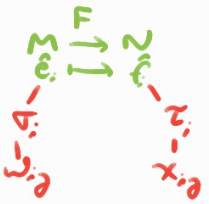
$F^*\Phi \longleftarrow \Phi$

$F^*\Phi(\vec{v}) = \Phi(F_* v)$

$$\Rightarrow \boxed{\begin{aligned} F^*(\Phi \wedge \Psi) &= F^*\Phi \wedge F^*\Psi \\ F^*(d\Phi) &= d(F^*\Phi) \end{aligned}}$$

Isometry

$$\boxed{\begin{aligned} \sigma_i &= F^*(\tau_i) \\ \omega_{ij} &= F^*(\chi_{ij}) \end{aligned}}$$



Pf: $\hat{f}_i = F_*(\hat{e}_i)$

$\Rightarrow F^*\tau_i(\hat{e}_j) = \tau_i(F_*\hat{e}_j) = \tau_i(\hat{f}_j) = \delta_{ij}$

$= \sigma_i(\hat{e}_j)$

$\Rightarrow F^*\tau_i = \sigma_i$ ✓

$d\tau_i = \chi_{ij} \wedge \tau_j \Rightarrow d F^*\tau_i = F^*\chi_{ij} \wedge F^*\tau_j$

$\Rightarrow d\sigma_i = F^*\chi_{ij} \wedge \sigma_j$

$= \omega_{ij} \wedge \sigma_j$

$\therefore F^*\chi_{ij} = \omega_{ij}$ ✓