

## Pullback

$$\begin{array}{ccc} F: M & \rightarrow & N \\ \downarrow & \mapsto & \downarrow \\ F_*v & & F_*v \\ F^*\phi & \longleftarrow & \phi \end{array}$$

$$F^*\phi(v) = \phi(F_*v)$$

$$\Rightarrow \begin{array}{l} F^*(\phi \wedge \psi) = F^*\phi \wedge F^*\psi \\ F^*(d\phi) = d(F^*\phi) \end{array}$$

## Isometry

$$\begin{array}{l} \sigma_i = F^*(\tau_i) \\ \omega_{ij} = F^*(\chi_{ij}) \end{array}$$

$$\begin{array}{ccc} M & \xrightarrow{F} & N \\ \downarrow & \mapsto & \downarrow \\ \hat{e}_i & & \hat{e}_j \\ \downarrow \sigma_i & & \downarrow \tau_i \\ \omega_{ij} & & \chi_{ij} \end{array}$$

Pf:  $\hat{f}_i = F_*(\hat{e}_i)$

$$\Rightarrow F^*\tau_i(\hat{e}_j) = \tau_i(F_*\hat{e}_j) = \tau_i(\hat{f}_j) = \delta_{ij} \\ = \sigma_i(\hat{e}_j)$$

$$\Rightarrow F^*\tau_i = \sigma_i \quad \checkmark$$

$$d\tau_i = \chi_{ij} \wedge \tau_j \Rightarrow dF^*\tau_i = F^*\chi_{ij} \wedge F^*\tau_j \\ \Rightarrow d\sigma_i = F^*\chi_{ij} \wedge \sigma_j \\ = \omega_{ij} \wedge \sigma_j$$

$$\therefore F^*\chi_{ij} = \omega_{ij} \quad \checkmark$$