§7.2:2

conformal structure with ruler function h

$$\Rightarrow \langle \vec{v}, \vec{w} \rangle = \frac{\vec{v} \cdot \vec{w}}{h^2}$$

$$\Rightarrow ds^2 = \frac{du^2 + dv^2}{h^2}$$

$$\Rightarrow \sigma_1 = \frac{du}{h}, \quad \sigma_2 = \frac{dv}{h}$$

- $h = h(uv) \Rightarrow u, v$ symmetry

$\Rightarrow$ "$u^2 - v^2$" impossible

§7.3:2

$\Leftrightarrow$: must show both directions

$\vec{w}$ parallel, $\langle \vec{u}, \vec{w} \rangle = \text{const}$

$$\Rightarrow \langle \nabla_{\vec{v}} \vec{u}, \vec{w} \rangle = -\langle \vec{u}, \nabla_{\vec{v}} \vec{w} \rangle = 0$$

$$\Rightarrow \nabla_{\vec{v}} \vec{u} \perp \vec{w}, \quad \underline{\text{not}} \quad \nabla_{\vec{v}} \vec{u} = 0$$

need: $|\vec{u}| = \text{const} \Rightarrow \nabla_{\vec{v}} \vec{u} \perp \vec{u}$

$\therefore$ done if $\vec{u}, \vec{w}$ indpt

if not, $\vec{u} = c\vec{w}$

can also argue using connection, but not necessary

§7.3:6

$$\omega_{12}(\vec{v}) = 0 \ \underline{\underline{\forall \vec{v}}} \Rightarrow \omega_{12} = 0 \Rightarrow d\omega_{12} = 0 \Rightarrow K = 0$$

"$d\omega_{12}(\vec{v})$", "$K(\vec{v})$" do not make sense

§7.4:8

what is α''??

2-d $\quad \alpha' \leftrightarrow \vec{v} \in TM$

$\alpha'' \leftrightarrow \nabla_{\vec{v}} \vec{v}$

$\mathbb{R}^3 \quad \bar{\alpha}' \leftrightarrow \vec{v} \in TM \Rightarrow \bar{\alpha}' = \alpha'$

$\bar{\alpha}'' \leftrightarrow \bar{\nabla}_{\vec{v}} \vec{v} = \frac{d\vec{v}}{dt} \Rightarrow \bar{\alpha}'' \neq \alpha''$

$\nabla_{\vec{v}} \vec{v} \neq \frac{d\vec{v}}{dt}$!

need $\quad \bar{\nabla}_{\vec{v}} \vec{v} = \nabla_{\vec{v}} \vec{v} + Q\hat{n}$

$$\Rightarrow \bar{\alpha}'' = \alpha'' + Q\hat{n}$$

$$\Rightarrow \bar{\alpha}' \times \bar{\alpha}'' = \alpha' \times \alpha'' + Q\alpha' \times \hat{n}$$

$$\Rightarrow \hat{n} \cdot \bar{\alpha}' \times \bar{\alpha}'' = \hat{n} \cdot \alpha' \times \alpha'' + Q\cancel{\hat{n} \cdot \alpha' \times \hat{n}}$$

$$\therefore \underline{\hat{n} \cdot \bar{\alpha}' \times \bar{\alpha}'' = \hat{n} \cdot \alpha' \times \alpha''}$$

Gauss-Bonnet Formula

simple closed curve

$\Rightarrow$ total geodesic curvature is

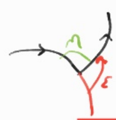
$$\oint_C k_g\,ds = \oint_C \omega_{12} + \oint_C d\varphi$$

$$= \int_R d\omega_{12} + 2\pi$$

$$= 2\pi - \int_R K\,dM$$

$$\therefore\ \boxed{\oint_{\partial R} k_g\,ds + \int_R K\,dM = 2\pi}$$

Polygons



$$\oint d\varphi = 2\pi - \sum \varepsilon_i$$

$$\therefore\ \boxed{\oint_{\partial R} k_g\,ds + \int_R K\,dM + \sum \varepsilon_i = 2\pi}$$

$$\boxed{\oint_{\partial R} k_g\,ds + \int_R K\,dM + \sum(\pi - \eta_i) = 2\pi}$$

Ex: rectangle $\sum(\pi - \eta_i) = 4\pi - \sum \eta_i$

triangle $\sum(\pi - \eta_i) = 3\pi - \sum \eta_i$

Geodesic Triangles

$$\oint_{\partial T} \cancel{k_g\, ds} + \int_T K\, dM + \underset{+3\pi}{\sum(\cancel{\pi} - \eta_i)} = 2\pi$$

$$\Rightarrow \underline{\sum \eta_i - \pi = \int K\, dM}$$

① plane

$$K = 0 \Rightarrow \sum \eta_i = \pi$$

angle sum

② sphere

$$K = 1/a^2 \Rightarrow \left(\sum \eta_i - \pi\right) a^2 = \text{area}$$

angle excess

③ pseudosphere / Poincaré Disk

$$K = -1/a^2 \Rightarrow \left(\pi - \sum \eta_i\right) a^2 = \text{area}$$

angle defect

∴ no similar triangles!