Dot & Cross Product

$$\vec{F} = F_i \hat{e}_i \longleftrightarrow F = F_i\, dx_i$$

$$\Rightarrow F \wedge G = F_i G_j\, dx_i \wedge dx_j$$

$$\Rightarrow \boxed{\vec{F} \times \vec{G} \longleftrightarrow *(F \wedge G)}$$

$$\& \Rightarrow F \wedge *G = (F_i G_i)\, dx \wedge dy \wedge dz$$

$$\Rightarrow \boxed{\vec{F} \cdot \vec{G} = *(F \wedge *G)}$$

$$\mapsto \vec{F} \cdot \vec{G} \times \vec{H} = *(F \wedge **(G \wedge H))$$
$$= *(F \wedge G \wedge H)$$

(triple product is cyclic)

Div, Grad, Curl

$$df = \frac{\partial f}{\partial x_i} dx_i$$

$$\Rightarrow \boxed{\vec{\nabla} f \longleftrightarrow df}$$

$$dF = \frac{\partial F_i}{\partial x_j} dx_j \wedge dx_i$$

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{F} \longleftrightarrow *\, dF}$$

$$d{*F} = \frac{\partial F_i}{\partial x_i} dx \wedge dy \wedge dz$$

$$\Rightarrow \boxed{\vec{\nabla} \cdot \vec{F} = *\, d * F}$$

$$\vec{\nabla} \times \vec{\nabla} f \longleftrightarrow *\, d\, df = 0$$

$$\vec{\nabla} \cdot \vec{\nabla} \times \vec{F} = *\, d * *\, dF$$
$$= *\, d\, dF = 0$$