

Darboux Frame

Assume $\vec{\alpha}$ is unit speed

$$\Rightarrow \hat{T} = \frac{d\vec{\alpha}}{ds}$$

$$\hat{n} \perp M$$

$$\hat{V} = \hat{n} \times \hat{T}$$

$$\Rightarrow \frac{d\hat{T}}{ds} = g\hat{V} + k\hat{n}$$

$$\frac{d\hat{V}}{ds} = -g\hat{T} + t\hat{n}$$

$$\frac{d\hat{n}}{ds} = -k\hat{T} - t\hat{V}$$

$$\Rightarrow k = k(\hat{T})$$

(use connection if $\vec{\alpha}$ not unit speed)

Special curves

$$t=0 \Leftrightarrow \text{principal}$$

$$k=0 \Leftrightarrow \text{asymptotic}$$

$$g=0 \Leftrightarrow \text{geodesic}$$