MTH 434/534 **HW #3** Winter 2024

1. **HODGE DUAL IN SPHERICAL COORDINATES**

Consider spherical coordinates in 3-dimensional Euclidean space with the usual orientation, namely $\omega = r^2 \sin\theta \, dr \wedge d\theta \wedge d\phi$.

WARNING: These are "physics" conventions: θ is the angle from the north pole (colatitude), and ϕ is the angle in the xy-plane (longitude).

(a) Determine the Hodge dual operator $*$ on all forms (expressed in spherical coordinates) by computing its action on basis forms at each rank.

(b) Compute the dot and cross products of two generic *1-forms* in spherical coordinates using the expressions:

$$\alpha \cdot \beta = *(\alpha \wedge *\beta)$$
$$\alpha \times \beta = *(\alpha \wedge \beta)$$

*You may express your results either with respect to an orthonormal basis or with respect to a "coordinate" (non-orthonormal) spherical basis; make sure you know which you're doing. ("Generic" means for **any** two 1-forms, e.g. in terms of their components.)*

2. **HODGE DUAL IN MINKOWSKI SPACE**

4-dimensional Minkowski space has an orthonormal, oriented basis of 1-forms given by

$$\{dx, dy, dz, dt\}$$

with $g(dt, dt) = -1$, $g(dx, dx) = g(dy, dy) = g(dz, dz) = 1$, and all others zero. The "volume element" (choice of orientation) is given by $\omega = dx \wedge dy \wedge dz \wedge dt$.

(a) Determine the Hodge dual operator $*$ on all forms by computing its action on basis forms at each rank.

(b) How does your answer change if the opposite orientation is chosen, namely

$$\omega = dt \wedge dx \wedge dy \wedge dz$$