USEFUL EQUATIONS: ($f \in \bigwedge^0$, $\alpha, \gamma \in \bigwedge^p$, $\beta \in \bigwedge^q$)

$$\beta \wedge \alpha = (-1)^{pq}\, \alpha \wedge \beta \qquad d(f\, d\alpha) = df \wedge d\alpha$$

$$d^2\alpha = 0 \qquad d(\alpha \wedge \beta) = d\alpha \wedge \beta + (-1)^p \alpha \wedge d\beta$$

$$\alpha \wedge *\gamma = g(\alpha, \gamma)\, \omega \qquad ** = (-1)^{p(n-p)+s}$$

$$d\hat{\boldsymbol{e}}_j = \omega^i{}_j\, \hat{\boldsymbol{e}}_i \qquad \omega_{ij} = \hat{\boldsymbol{e}}_i \cdot d\hat{\boldsymbol{e}}_j \qquad \omega_{ij} + \omega_{ji} = 0$$

$$d\vec{\boldsymbol{r}} = \sigma^i\, \hat{\boldsymbol{e}}_i \qquad d^2\vec{\boldsymbol{r}} = \vec{\boldsymbol{0}} \qquad d^2\hat{\boldsymbol{e}}_j = \Omega^i{}_j\, \hat{\boldsymbol{e}}_i$$

$$d\sigma^i + \omega^i{}_j \wedge \sigma^j = 0 \qquad \omega^i{}_j = \Gamma^i{}_{jk}\, \sigma^k$$

$$d\omega^i{}_j + \omega^i{}_k \wedge \omega^k{}_j = \Omega^i{}_j = \frac{1}{2}\, R^i{}_{jkl}\, \sigma^k \wedge \sigma^l$$

$$\Omega^i{}_j \wedge \sigma^j = 0 \qquad d\Omega^i{}_j + \omega^i{}_k \wedge \Omega^k{}_j - \Omega^i{}_k \wedge \omega^k{}_j = 0$$

$$S = -\left(\Gamma^3{}_{ij}\right) \qquad \det S = K \qquad \Omega^1{}_2 = d\omega^1{}_2 = K\, \sigma^1 \wedge \sigma^2$$

$$\hat{\boldsymbol{T}} = \cos\alpha\, \hat{\boldsymbol{e}}_1 + \sin\alpha\, \hat{\boldsymbol{e}}_2 \qquad \kappa_g\, ds = d\hat{\boldsymbol{T}} \cdot \hat{\boldsymbol{N}} = d\alpha - \omega^1{}_2 \qquad \int_S K\, \omega + \oint_{\partial S} \kappa_g\, ds = 2\pi$$

$$\int_R d\alpha = \int_{\partial R} \alpha \qquad \int_\Sigma K\, \omega = 2\pi\chi(\Sigma) = 2\pi(v - e + f)$$

$$2\sin^2\theta = 1 - \cos 2\theta \qquad \frac{d}{d\theta} \ln\tan\frac{\theta}{2} = \frac{1}{\sin\theta}$$

$$\cosh^2\psi - \sinh^2\psi = 1 \qquad 2\cosh\psi = e^\psi + e^{-\psi} \qquad 2\sinh\psi = e^\psi - e^{-\psi}$$

$$d\sinh\psi = \cosh\psi\, d\psi \qquad d\cosh\psi = \sinh\psi\, d\psi$$

You may wish to use the following relationships in (Euclidean) $\mathbb{R}^3$:

$$\vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}} = F$$

$$\vec{\nabla} f \cdot d\vec{\boldsymbol{r}} = df$$

$$(\vec{\nabla} \times \vec{\boldsymbol{F}}) \cdot d\vec{\boldsymbol{r}} = *dF \qquad (\vec{\boldsymbol{F}} \times \vec{\boldsymbol{G}}) \cdot d\vec{\boldsymbol{r}} = *(F \wedge G)$$

$$\vec{\nabla} \cdot \vec{\boldsymbol{F}} = *d{*F} \qquad \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{G}} = *(F \wedge *G)$$

$$\triangle f = \vec{\nabla} \cdot \vec{\nabla} f = *d{*df}$$