MTH 434/534 **HW #2** Winter 2024

1. **DECOMPOSABLE FORMS**

Denote the $p$-forms in $\mathbb{R}^n$ by $\bigwedge^p(\mathbb{R}^n)$. A typical 1-form in $\mathbb{R}^2$ would therefore take the form $F = F_x\,dx + F_y\,dy \in \bigwedge^1(\mathbb{R}^2)$.

A $p$-form $\beta \in \bigwedge^p(\mathbb{R}^n)$ is called *decomposable* if there exist 1-forms $\alpha_i \in \bigwedge^1(\mathbb{R}^n)$ with

$$\beta = \alpha_1 \wedge ... \wedge \alpha_p$$

(a) Show that all elements of $\bigwedge^2(\mathbb{R}^3)$, that is, all 2-forms in $\mathbb{R}^3$, are decomposable. In other words, show that

$$H = H_x\,dy \wedge dz + H_y\,dz \wedge dx + H_z\,dx \wedge dy$$

is decomposable.

*HINT: Consider the previous assignment!*

***You may cite your solution to the previous assignment without proof, so long as an explicit reference is given ("see HW #1"). If you do this, it wouldn't hurt to include a copy of your previous assignment.***

(b) Find an example of an *indecomposable* 2-form $\gamma \in \bigwedge^p(\mathbb{R}^n)$.

*HINT: Don't work in* $\mathbb{R}^3$*...*

(c) Is $\gamma \wedge \gamma = 0$? Should it be? Can it be?

(d) ***MTH 534 students only (or extra credit):***

Show that all 3-forms are decomposable in $\mathbb{R}^4$.

***Extra Credit:*** Can you argue that all elements of $\bigwedge^{n-1}(\mathbb{R}^n)$ are decomposable?

2. **PICTURES OF FORMS**

Let $\alpha = 3\,dx$ and $\beta = 4\,dy$.

(a) Draw a single picture showing the "stacks" corresponding to both $\alpha$ and $\beta$.

*You may want to use different colors for the stacks corresponding to $\alpha$ and $\beta$. Your drawing should be correctly scaled.*

(b) Draw a separate picture showing the stack corresponding to $\gamma = \alpha + \beta$.

(c) Choose a vector $\vec{\boldsymbol{v}} \in \mathbb{R}^2$ that is *not* parallel to the coordinate axes. Add $\vec{\boldsymbol{v}}$ to your previous diagrams and use them to compute $\alpha(\vec{\boldsymbol{v}})$, $\beta(\vec{\boldsymbol{v}})$, and $\gamma(\vec{\boldsymbol{v}})$.

*Your computation should be geometric, not algebraic.*

(d) Did you obtain $\gamma(\vec{\boldsymbol{v}}) = \alpha(\vec{\boldsymbol{v}}) + \beta(\vec{\boldsymbol{v}})$? Should you have?