$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z} \qquad x = x(u,v,w)$$

$$d\vec{r} = \frac{\partial \vec{r}}{\partial u}du + \cdots \qquad \vdots$$

$$\sigma\cdot\hat{e} = \sigma^u \hat{u} + \underline{\qquad\quad}$$

$\Longrightarrow$ orthonormal basis of 1-forms:

$$\sigma^u = h_u\, du \quad \text{with}$$

$$h_u = \left|\frac{\partial \vec{r}}{\partial u}\right|$$

Line Elements

$$x = \tfrac{1}{2}(u^2 - v^2)$$
$$y = uv$$

Zap w/d :

$$dx = u\,du - v\,dv$$
$$dy = v\,du + u\,dv$$

$$\Rightarrow ds^2 = dx^2 + dy^2$$
$$= (\qquad)^2 + (\qquad)^2$$
$$= (u^2+v^2)\,du^2 + 0\,du\,dv + (u^2+v^2)\,dv^2$$
$$= (u^2+v^2)(du^2+dv^2)$$

$$\Rightarrow \sigma^u = \sqrt{u^2+v^2}\,du$$
$$\sigma^v = \sqrt{u^2+v^2}\,dv \Rightarrow \omega = \sigma^u \wedge \sigma^v$$
$$= (u^2+v^2)\,du \wedge dv$$

$$\Rightarrow *\left(\sqrt{u^2+v^2}\,du\right) = \sqrt{u^2+v^2}\,dv$$

$$\Rightarrow *du = dv \quad !$$

$$\vec{r} = x\hat{x} + y\hat{y}$$
$$\frac{\partial \vec{r}}{\partial u} = u\hat{x} + v\hat{y}$$
$$\frac{\partial \vec{r}}{\partial v} = -v\hat{x} + u\hat{y}$$

$$x = \tfrac{1}{2}(u^2 - v^2)$$
$$y = uv$$