Integration

Recall: $f \in \Lambda^0(\mathbb{R}^3)$, $F \in \Lambda^1(\mathbb{R}^3)$

$\Rightarrow$ $*f = f\,dV$ $\qquad \nabla f = df = \vec{\nabla} f \cdot d\vec{r}$

$F = \vec{F} \cdot d\vec{r}$ $\qquad \nabla \times F = * dF = \vec{\nabla} \times \vec{F} \cdot d\vec{r}$

$*F = \vec{F} \cdot d\vec{A}$ $\qquad \nabla \cdot F = * d * F = \vec{\nabla} \cdot \vec{F}$

$\Rightarrow$ $df = \vec{\nabla} f \cdot d\vec{r}$

$dF = \vec{\nabla} \times \vec{F} \cdot d\vec{A}$

$d*F = \vec{\nabla} \cdot \vec{F}\, dV$

FT for gradient: $\int_C \vec{\nabla} f \cdot d\vec{r} = f\Big|_A^B$ $\qquad \int_C df = \int_{\partial C} f$

Stokes' Thm: $\int_S \vec{\nabla} \times \vec{F} \cdot d\vec{A} = \oint_C \vec{F} \cdot d\vec{r}$ $\qquad \int_S dF = \int_{\partial S} F$

Divergence Thm: $\int_V \vec{\nabla} \cdot \vec{F}\, dV = \oint_S \vec{F} \cdot d\vec{A}$ $\qquad \int_V d*F = \int_{\partial V} *F$

$$\therefore \quad \boxed{\int_R d\alpha = \int_{\partial R} \alpha}$$

Stokes' Theorem

"outward" orientation of boundary is tricky if $s \neq 0$

for any $\alpha \in \Lambda^p$