HW #5

1. **INTEGRATION ON THE SPHERE**

Consider $\mathbb{S}^2$, which can be viewed as the surface in $\mathbb{E}^3$ satisfying $x^2 + y^2 + z^2 =$ constant. Equivalently, it is the 2-dimensional surface with line element $ds^2 = r^2(d\theta^2 + \sin^2\theta \, d\phi^2)$.

(a) Let ω be the orientation of $\mathbb{S}^2$. Determine $\displaystyle\int_{\mathbb{S}^2} \omega$.

(b) Let α be any 1-form on $\mathbb{S}^2$, that is, $\alpha \in \bigwedge^1(\mathbb{S}^2)$. Use Stokes' Theorem to compute $\displaystyle\int_{\mathbb{S}^2} d\alpha$.

(c) Find a 1-form on $\mathbb{S}^2$ such that $d\alpha = \omega$.

(d) How is this possible?

You may wish to start by considering the analogous problem on the circle.

You may translate this problem into the language of vector calculus, but you should then (also) clearly indicate how to solve the problem in the language of differential forms.